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# Second Order Numerical Iterated Method of Newton-Type for Estimating a Single Root of Nonlinear Equations

Umair Khalid Qureshi[1], Zubair Ahmed kalhoro[2]

[1]Department of Basic Science & Related Studies, Mehran University of Engineering and technology, Pakistan
[2]Institute Mathematics and Computer Science, University of Sindh Jamshoro, Pakistan
Email: khalidumair531@gmail.com, zubair.kalhoro@usindh.edu.pk

***Abstract:*** This study deals with the Newton-type numerical method to estimating a single root of nonlinear equations. The proposed method is converged quadratically and based on a Newton Raphson Method and step-size. Numerical tests demonstration of developed technique with well-known Steffensen Method and Newton Raphson Method. It is experiential from fallouts and comparisons of developed method is that the Second Order Newton-Type Iterated Method is superior than existing second order methods.

**Keywords:** Nonlinear problems, Newton’s Technique, Quadratic Technique, Convergence Analysis, Accuracy.

## I. INTRODUCTION

The most commonly applications problems in engineering and applied Sciences [1], [3] and [4] are estimating nonlinear problem, such as

$$f(x) = 0 \tag{1}$$

Excluding rare cases, the solution Eq. (1) cannot be resolved in straight way. For that reason, to solving Eq. (1) has used numerical iterative methods, such as most solvable iterative method is Newton’s method. The Newton’s method defined as

$$x_{n+1} = x_n - \frac{f(x_n)}{f`(x_n)} \tag{2}$$

This method is fast converging and second quadratically converge technique but are not reliable because keeping pitfall [5] and [6]. However, it is most useful and vigorous numerical techniques. Several modifications in Newton Raphson techniques had been done by using Taylor series and difference operator to be estimating nonlinear equations in literatures [2], [7], [8], [9], [10], [11] and [12].

Similarly, this study has used the Newton’s technique to construct a newton-type numerical algorithm. For using newton technique, firstly, we use the mid-point in newton raphson method then we use the step-size. Now, we can construct a new iterative method for solving the nonlinear equations. We are also proven convergence of the proposed method, which is order of two. Few numerical examples are given to the exemplify the efficiency and the performance of new iterative method. Our results can be considered as an important improvement and refinement of the previously results.

## II. NUMERICAL METHOD

By using the Newton Raphson Method, such as

$$x = x_0 - \frac{f(x_0)}{f`(x_0)}$$

For better approximation, we are taking average of $\frac{1}{f`(x)}$ at point $(x_0, x_1)$, we get

$$x = x_0 - \frac{f(x_0)}{2}\left[\frac{1}{f`(x_0)} + \frac{1}{f`(x_1)}\right]$$

In general,

$$x = x_n - \frac{f(x_n)}{2}\left[\frac{1}{f`(x_n)} + \frac{1}{f`(x_{n+1})}\right]$$

Where $x_0$ is an initial guess sufficiently close to root, to modify the above equation by using step-size, such as

$$h = x_{n+1} - x_n$$

or

$$x_{n+1} = x_n + h$$

By using numerical condition that is $h = \Delta(x_n) = f(x_n)$ in above, then substitute in above, we get

$$x_{n+1} = x_n + f(x_n)$$

Finally, above condition substitute in general equation, we get

$$x_{n+1} = x_n - \frac{f(x_n)}{2}\left[\frac{1}{f`(x_n)} + \frac{1}{f`(x_n + f(x_n))}\right]$$

Hence this is the new iterated method.

## III. RATE OF CONVERGENCE

The following section will be shows that the New Developed Method is Quadratic Convergence.

**proof**

Using the relation $e_n = x_n - a$ in Taylor series, therefore from Taylor series, we estimate $f(x_n), f`(x_n)$ *and* $f`(x_n + f(x_n))$ with using this condition $c = \frac{f``(a)}{2f`(a)}$ and ignoring higher order term, we have

$$f(x_n) = f`(a)(e_n + ce_n{}^2) \qquad --- \ (i)$$

$$f`(x_n) = f`(a)(1 + 2ce_n) \qquad --- \ (ii)$$

and

$$f(x_n + f(x_n)) = f`(a)[(e_n + f(x_n)) + c(e_n + f(x_n))^2$$

Or

$$f`(x_n + f(x_n)) = f`(a)[(1 + f`(x_n)) + 2c(1 + f`(x_n))(e_n + f(x_n))]$$

$$f`(x_n + f(x_n)) = f`(a)(1 + f`(x_n))\,[1 + 2c\,(e_n + f(x_n))] \qquad --- \ (iii)$$

*By using* Eq. $(i)$ *and* Eq. $(ii)$ *in* Eq. $(iii)$,

$$f`(x_n + f(x_n)) = f`(a)(1 + f`(a)(1 + 2ce_n))\,[1 + 2ce_n(1 + f`(a))]$$

$$f`(x_n + f(x_n)) = f`(a)[1 + f`(a) + 2ce_nf`(a) + 2ce_n + 2ce_nf`(a) + 2ce_nf`^2(a) + 2ce_nf`(a)] \qquad --- \ (iv)$$

Eq. $(i)$, Eq. $(ii)$ *and* Eq. $(iv)$ substitute in developed method, we get

$$e_{n+1} = e_n - \frac{f`(a)e_n(1 + ce_n)}{2f`(a)}\left[\frac{1}{1 + 2ce_n} + \frac{1}{1 + f`(a) + 2ce_n[1 + 3f`(a) + f`^2(a)]}\right]$$

*or*

$$e_{n+1} = e_n - \frac{e_n(1 + ce_n)}{2}[1 - 2ce_n + 1 - f`(a) - 2ce_n[1 + 3f`(a) + f`^2(a)]]$$

$$e_{n+1} = e_n - \frac{e_n(1 + ce_n)}{2}[2 - f`(a) - 2ce_n[2 + 3f`(a) + f`^2(a)]]$$

$$e_{n+1} = e_n - e_n(1 + ce_n)[1 - \frac{f`(a)}{2} - ce_n[2 + 3f`(a) + f`^2(a)]]$$

$$e_{n+1} = e_n - e_n(1 - \frac{f`(a)}{2} - ce_n[2 + 3f`(a) + f`^2(a)] + ce_n - ce_n\frac{f`(a)}{2})$$

$$e_{n+1} = \frac{e_nf`(a)}{2} + ce_n{}^2\left[3 + \frac{5f`(a)}{2} + f`^2(a)\right] \qquad --- \ (v)$$

For using Eq. (1) inEq. $(i)$then substitute in Eq. (v), we obtain

$$e_{n+1} = \frac{-f``(a)e_n{}^2}{2} + ce_n{}^2\left[3 + \frac{5f`(a)}{2} + f`^2(a)\right]$$

$$e_{n+1} = e_n{}^2\left[\frac{-f``(a)}{2} + c\left(3 + \frac{5f`(a)}{2} + f`^2(a)\right)\right]$$

Hence this proves that the proposed iterative method has second order of convergence.

## IV. RESULTS AND DISCUSSIONS

This session has been inspected few of functions in Table-1. The results had examined with Newton Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f`(x_n)}$$

and Steffensen Method

$$x_{n+1} = x_n - \frac{f^2(x_n)}{f\big(x_n + f(x_n)\big) - f(x_n)}$$

All calculation has been completed by using C++ programing or MATLAB and with stopping criteria $|x_{n+1} - x_n| < 10^7$. In Table-1, we have listed the fallouts obtained by new Newton-type Method and compared with Steffensen Method and Newton Raphson Method and from outcomes it has been observed that proposed method takes less number of iterations and gives better accuracy. Henceforth, from Table-1 it is evidently demonstration that the result obtained by proposed method is most effective as it faster convergence and iterations viewpoint with the assessment of Steffensen Method and Newton Raphson Method for solving nonlinear equations.

Table 1: Fallouts Obtained

| FUNCTIONS | METHODS | ITERATIONS | X | A E % |
|---|---|---|---|---|
| **$2x^2$ -5x-2** **$x_0$=0** | Steffensen Method | 6 | 0.350781 | 1.19209 $e^{-6}$ |
| | Newton Raphson Method | 5 | | 2.98023$e^{-8}$ |
| | Newton-type Method | 3 | | 1.15931$e^{-5}$ |
| **2x-lnx-7** **$x_0$=6** | Steffensen Method | 4 | 4.21991 | 1.58311$e^{-4}$ |
| | Newton Raphson Method | 4 | | 4.76837$e^{-5}$ |
| | Newton-type Method | 3 | | 1.37329$e^{-4}$ |
| **$\sin^2$x-$x^2$+1** **$x_0$=1** | Steffensen Method | 5 | 1.40449 | 1.10031$e^{-4}$ |
| | Newton Raphson Method | 5 | | 5.96046$e^{-7}$ |
| | Newton-type Method | 4 | | 1.44243$e^{-5}$ |
| **$e^{-x}$-cosx** **$x_0$=4** | Steffensen Method | 3 | 4.72129 | 4.42076$e^{-3}$ |
| | Newton Raphson Method | 3 | | 1.21212$e^{-3}$ |
| | Newton-type Method | 3 | | 1.57356$e^{-5}$ |
| **Sinx-x+1** **$x_0$=1** | Steffensen Method | 4 | 1.93456 | 3.18646$e^{-4}$ |
| | Newton Raphson Method | 5 | | 5.84126$e^{-6}$ |
| | Newton-type Method | 4 | | 6.85453$e^{-5}$ |

## V. CONCLUSION

This paper has composed a Newton-type numerical iterative method with second order convergence obtained with Newton raphson method and numerical technique for estimating a root of nonlinear equations. From Table-1, it has been shown that the efficiency of proposed method performs better than classical second order methods as iteration point of view and as well as accuracy perceived. Henceforth, from numerical results demonstrate that the proposed method is performing-well, more efficient and easy to employ with reliable results for solving non-linear equations.

## VI. REFERENCES

[1]. Biswa N. D. 2012, Lecture Notes on Numerical Solution of root Finding Problems.

[2]. Qureshi, U. K., Kalhoro, Z. A., Bhutto, G. Y., Khokar, R. B., & Qureshi, Z. A. (2018). Modified Linear Convergence Mean Methods for Solving Non-Linear Equations. University of Sindh Journal of Information and Communication Technology, 2(1), 31-35.

[3]. Yasmin, N. and M.U.D. Janjua 2012, Some Derivative Free Iterative Methods for Solving Nonlinear Equations, ISSN-L: 2223 9553, ISSN: 2223-9944, Vol. 2, No.1. 75-82.

[4]. Iwetan, C. N. 2012, Comparative Study of the Bisection and Newton Methods in solving for Zero and Extremes of a Single-Variable Function. J. of NAMP Vol.21 173-176.

[5]. Akram, S. and Q. U. Ann 2015, Newton Raphson Method, International Journal of Scientific & Engineering Research, Volume 6.

[6]. Soram R. 2013, On the Rate of Convergence of Newton-

Raphson Method, The International Journal of Engineering and Science, Vol-2, ISSN(e): 2319 – 1813 ISSN(p): 2319 – 1805.

[7]. Qureshi, U. K., Kalhoro, Z. A., Shaikh, A. A., & Nangraj, A. R. (2018). Trapezoidal Second Order Convergence Method for Solving Nonlinear Problems. University of Sindh Journal of Information and Communication Technology, 2(2), 111-114.

[8]. Qureshi, U. K. 2017, Modified Free Derivative Open Method for Solving Non-Linear Equations, Sindh University Research Journal, Vol.49 (004) 821-824 (2017).

[9]. Allame M. and N. Azad 2012, On Modified Newton Method for Solving a Nonlinear Algebraic Equations by Mid-Point, World Applied Sciences Journal Vol.17(12): 1546-1548, ISSN 1818-4952 IDOSI Publications.

[10]. Omran, H. H. 2013, Modified Third Order Iterative Method for Solving Nonlinear Equations, Journal of Al-Nahrain University, Vol.16 (3), pp.239-245

[11]. Soomro, E. 2016, On the Development of a New Multi-Step Derivative Free Method to Accelerate the Convergence of Bracketing Methods for Solving, Sindh University Research Journal (Sci. Ser.) Vol. 48(3) 601-604.

[12]. Qureshi, U. K. 2017, Modified Free Derivative Open Method for Solving Non-Linear Equations, Sindh University Research Journal, Vol.49 (004) 821-824 (2017).