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Trapezoidal Second Order Iterated Method for Solving Nonlinear Problems

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Abstract: This study has been discussed a trapezoidal iterated method for estimating a single root of nonlinear problems, which arises in many scientific and engineering fields. The proposed iterative method has second order of convergence, and derived from trapezoidal rule. Several numerical examples to illustrate the efficiency of proposed method by Comparison with the famous second order convergence Newton Raphson Method. C++ and EXCELL are used to analyze the results and graphical representations of proposed method. Henceforth, it has been observed from the outcomes and assessment of trapezoidal iterative method with the Newton Raphson Method that the trapezoidal second order method is performance healthier than existing method.

Keywords: non-linear problems, trapezoidal rule, order of convergence, absolute percentage error, accuracy.

I. INTRODUCTION

Solving nonlinear problems are one of the most attracted part of study in numerical inspection, which is attentive applications in pure and applied science [1], [2] and [3], such as nonlinear problems

$$f(x) = 0 \qquad ---(1)$$

For solving Eq. (1), numerous numerical techniques under certain conditions are recommended in literature. Such as, most commonly and useful bracketing techniques includes Bisection and False position techniques [4]. They are useful and modest numerical techniques. On the other hand, Newton Raphson Method are fast converging numerical techniques but are not reliable because keeping a kind of drawback [5]. In similar manner, recently deprived of derivative methods have established, which is modest, calmer to practice and deprived of downside [6] and [7].

Actually, two significant characteristics in iterative techniques are order of convergence and computational efficiency. Numerous iterative techniques have been constructive for order of convergence and computational efficacy by using different techniques such as finite difference, Taylor series, homotopy perturbation method and its variant forms, quadrature formula, and decomposition method by [8], [9], [10], [11], [12] and [13]. In the bright of overhead examination, we have suggested a new iterated method for estimating a single root of nonlinear equations by using trapezoidal rule. The proposed method has a second order of convergence. The proposed method is fast converging as contest on Newton Raphson Method for solving non-linear problems.

II. PROPOSED METHOD

By using trapezoidal rule and midpoint method instead of rectangular method to developed a Trapezoidal Second Order Convergence Method, such as

$$\int_{x_0}^{x} f`(x)dx = \frac{x - x_0}{2}[f`(x_0) + f`(x_1)] \qquad (2)$$

the placement of certain integration, we get the following statement,

$$f(x) = f(x_0) + \frac{x - x_0}{2}[f`(x_0) + f`(x_1)] \qquad (3)$$

according to Eq. (1), then

$$f(x_0) + \frac{x - x_0}{2}[f`(x_0) + f`(x_1) = 0 \qquad (4)$$

Further solving Eq. (4), we obtain

$$x = x_0 - \frac{2f(x_0)}{f`(x_0) + f`(x_1)} \qquad (5)$$

In general,

$$x_{n+1} = x_n - \frac{2f(x_n)}{f`(x_n) + f`(x_{n+1})} \text{(6)}$$

For using numerical condition, we have

$$x_{n+1} = x_n + h \qquad (7)$$

Where h= $\Delta(x_n)$=f(x_n) is using Eq. (7), then Eq. (7) substitute in Eq. (6), finally we get

$$x_{n+1} = x_n - \frac{2f(x_n)}{f`(x_n) + f`\big(x_n + f(x_n)\big)} \text{(8)}$$

This relationship is a Trapezoidal Iterated Method.

III. RATE OF CONVERGENCE

The following section will be shows that the Trapezoidal Iterated Method is Quadratic Convergence.

proof

Using the relation$e_n = x_n - \mathrm{a}$ in Taylor series,we are estimating $f(x_n), f`(x_n)$and $f`\big(x_{n+1} + f(x_{n+1})\big)$ byusing condition c = $f``(a)/2f`(a)$ and ignoring higher order term, therefore

$$f(x_n) = f`(a)(e_n + ce_n{}^2) \qquad --- \ (9)$$

$$f`(x_n) = f`(a)(1 + 2ce_n) \qquad --- \ (10)$$

and

$$f`\big(x_n + f(x_n)\big) = f`(a)\big(1 + f`(x_n)\big)\,[1 + 2c\left(e_n + f(x_n)\right)] \quad --- \ (11)$$

Substitute Eq. (9) and Eq. (10) in Eq. (11), then

$$f`\big(x_n + f(x_n)\big) = f`(a)\ [1 + f`(a) + 2ce_n[1 + 3f`(a) + f`^2(a)]] \quad --- (12)$$

AddingEq. (10) and Eq. (12),

$$f`\big(x_n + f(x_n)\big) + f`(x_n) = f`(a)[2 + f`(a) + 2ce_n(1 + 3f`(a) + f`^2(a))] \quad --- (13)$$

By using Eq. (9) and Eq. (13) in Eq. (8), we get

$$e_{n+1} = e_n - \frac{2f`(a)(e_n + ce_n{}^2)}{f`(a)[2 + f`(a) + 2ce_n(1 + 3f`(a) + f`^2(a))]}$$

$$e_{n+1} = e_n - \frac{e_n(1 + ce_n)}{\left[1 + \frac{f`(a) + 2ce_n(1+3f`(a)+f`^2(a))}{2}\right]}$$

$$e_{n+1} = e_n - e_n(1 + ce_n)\left[1 + \frac{f`(a) + 2ce_n(1 + 3f`(a) + f`^2(a))}{2}\right]^{-1}$$

$$e_{n+1} = e_n - e_n(1 + ce_n)\left[1 - \frac{f`(a) + 2ce_n(1 + 3f`(a) + f`^2(a))}{2}\right]$$

$$e_{n+1} = \left[\frac{e_n f`(a)}{2} + ce_n{}^2\left(2 + \frac{5f`(a)}{2} + f`^2(a)\right)\right] \quad --- (14)$$

for nonlinear equation such as$f(x_n) = 0$ in usingEq. (9) then Eq. (9) substitute in Eq. (14), we get

$$e_{n+1} = \left[\frac{-e_n{}^2 f``(a)}{4} + ce_n{}^2\left(2 + \frac{5f`(a)}{2} + f`^2(a)\right)\right]$$

$$e_{n+1} = e_n{}^2\left[\frac{-f``(a)}{4} + c(2 + \frac{5f`(a)}{2} + f`^2(a))\right]$$

Henceforth,this has been proven that the proposed trapezoidal iterative method is converge quadratically.

IV. RESULTS AND DISCUSSIONS

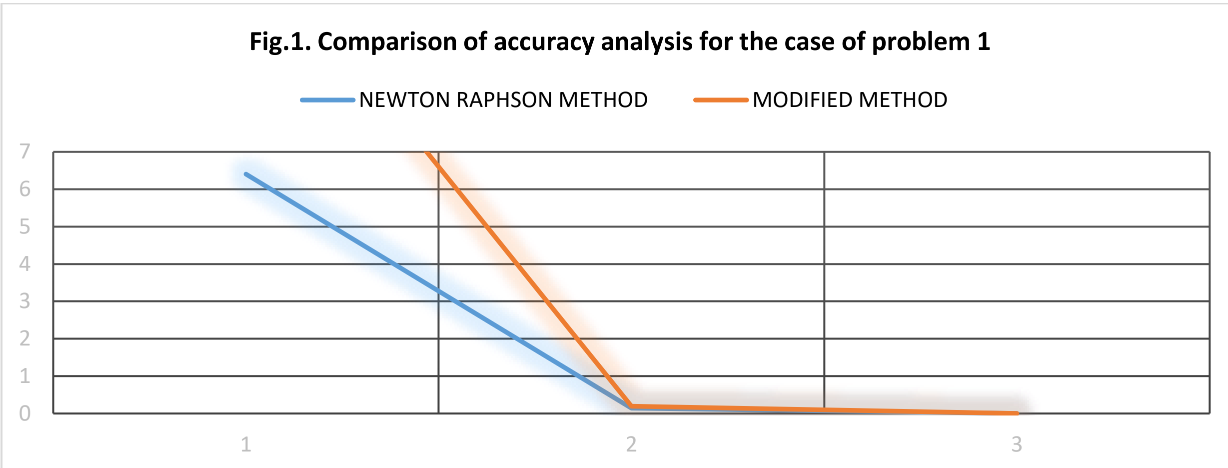
In this section, the new second order iterative method is applied on few examples of nonlinear application problems i.e. Distance, rate, time problem, population change, Trajectory of a ball, etc. To justify the results and graphical representations of proposed second order iterative method by using C++ and EXCEL. From the results, it has been percieved that developedtechnique is better than Newton Raphson Method. The proposed method compared with Newton Raphson Method, such as in Table-1 and figures as below

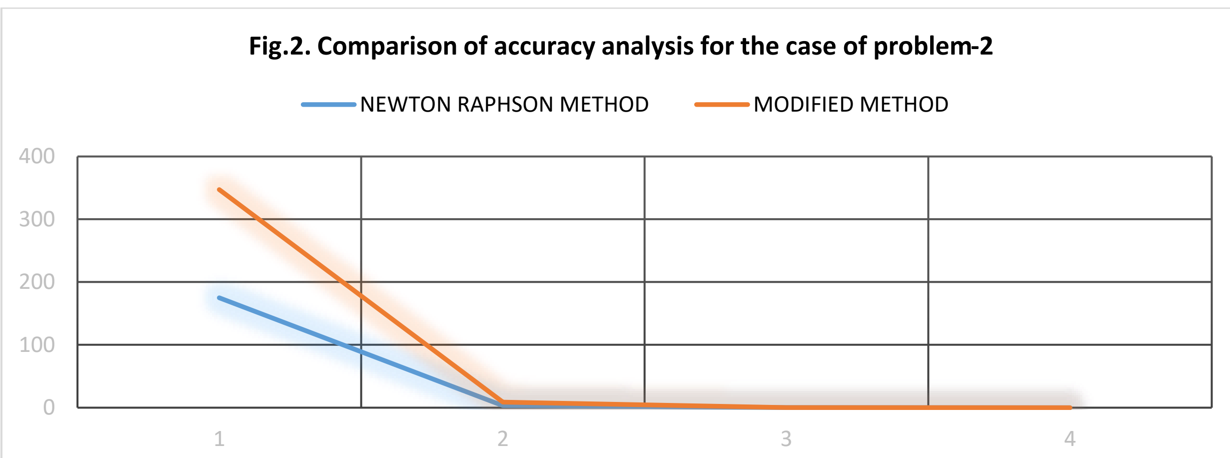
TABLE-1

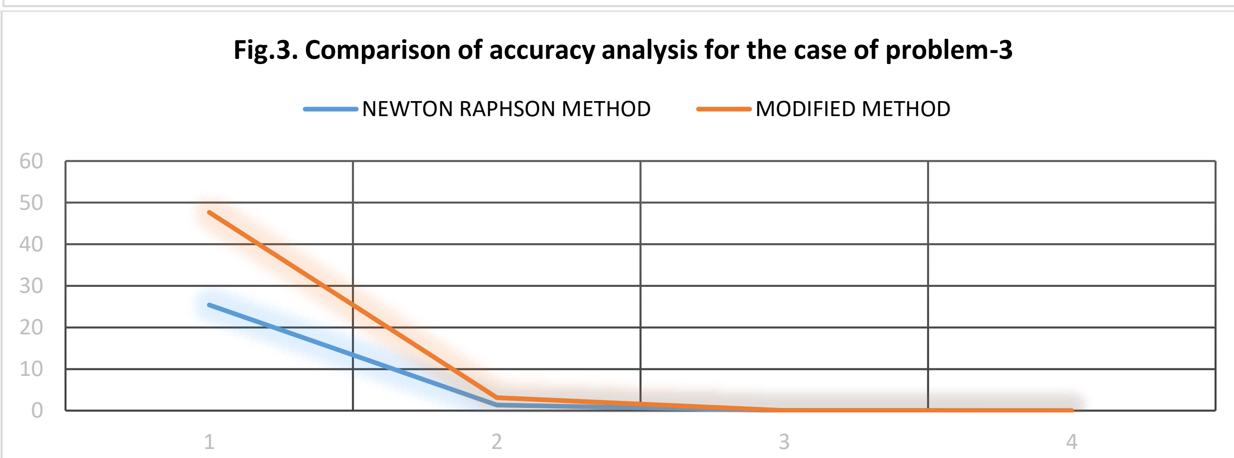
FUNCTIONS	METHODS	ITERATIONS	X	A E %
Sinx-x+1 x_0=2	Newton Raphson Method New Method	3 2	1.93456	$7.15256e^{-5}$ $5.71370e^{-2}$
2x-lnx-7 x_0=6	Newton Raphson Method New Method	4 3	4.21991	$4.76837e^{-5}$ $1.39236e^{-2}$
e^x–5x x_0=0.5	Newton Raphson Method New Method	4 3	0.259171	$2.98023e^{-6}$ $1.51277e^{-2}$
x^2 -2x-5 x_0=0	Newton Raphson Method New Method	5 4	1.44949	$4.64916e^{-4}$ $2.51651e^{-2}$

Cosx-x x_0=3	Newton Raphson Method New Method	6 5	0.739085	5.96046e^{-6} 4.76837e^{-5}
4sinx-e^x x_0=1	Newton Raphson Method New Method	6 5	1.36496	4.02927e^{-3} 4.68493e^{-3}
e^{-x}-cosx x_0=5.5	Newton Raphson Method New Method	4 3	4.72129	4.76837e^{-5} 1.19209e^{-3}

FIGURES







V. CONCLUSION

This study a trapezoidal second order iterated method has been presented for solving nonlinear problems by using trapezoidal rule. Through the study, it has been concluded that the proposed method is supercilious than Newton Raphson Method from iterative viewpoint as well as accuracy perspective. Hence, the proposed trapezoidal second order iterated method is fast converging and seems to be very easy for resolving non-linear problems.

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