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Modified Linear Convergence Mean Methods for Solving Non-Linear Equations

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Abstract In this paper, we have suggested a Modified Mean Methods for solving non-linear equations. Proposed Methods have linear order of convergence. The proposed Modified Mean Methods are used to solve all possible roots of non-linear functions in simpler and easier way. The proposed Methods are working tremendous as compare to Bisection method based on iterations and accuracy. To examine the fallouts of few problems, which are related by the non-linear functions to observe the efficiency of develop Modified Mean Methods. C++ and EXCEL have been used for obtaining results and graphical representations to justify the proposed methods. Throughout the study, it has been observed that the developed Modified Mean Algorithms are better techniques for estimating a root of non-linear equations.

Keyword: Non-linear equations, bisection method, convergence analysis, accuracy.

I. INTRODUCTION

During many years' various scientist and researchers have taken interest and given multiple methods for solving non-linear equations. The root-finding problem is one of the most relevant computational problems. It arises in a wide variety of practical applications in Physics, Chemistry, Biosciences, Engineering, etc. As a matter of fact, the determination of any unknown appearing implicitly in scientific or engineering formulas, gives rise to root finding problem (Datta, 2012) and (Iwetan,2012). Due to their importance; several methods have been suggested and analyzed under certain conditions. One of the easiest root-locating methods is Bisection method. It works when F(x) is a continuous function and it needs preceding information of two initial guesses, a and b, that is f(a) and f(b) have opposite signs, then estimate the average of $[a,b]$, and then choose whether the root lies on $[a, \frac{a+b}{2}]$ or $[\frac{a+b}{2}, b]$. Repeat as long as the interval is sufficiently small. (Dalquist and Bjorck, 2008) and (Mathews and Fink, 2004) have recommended that a root `x` exists in (a,b) with f(x) = 0. Even though the Bisection method is consistent, but it converges very slowly. Researchers have tried and are trying to increase the convergence rate of Bisection method through different techniques by (Chitra et al, 2014) and (Tanakan, 2013). Moreover, one of the fast converging numerical methods is Newton Raphson Method. It is more effective than the Bisection method, but sometimes Newton Raphson method fails to locate the roots. However, once it converges, it converges quicker than Bisection method. Combining these methods for better accuracy and less number of iteration is observed in number of articles.(Masood Allame &Nafiseh Azad, 2012) and (Mcdougall & Wotherspoon, 2013) are using Newton Raphson Method and Bisection method to construct a new iterated method, which is more quickly convergence than Newton raphson method, hybrid or new hybrid iterated method and midpoint Newton raphson method. Furthermore, Bisection Method somewhat called Arithmetic Mean between a & b. Recently researchers took interest and given good method, for instance (Aruchunanet al, 2015) has presented a Modified Arithmetic Mean technique for ordinary Arithmetic Mean technique for the solution of fourth order Fredholm Integro-differential equations, and the execution of Arithmetic Mean technique for estimating dense linear system relatedby numerical solution by (Muthuvalu and Sulaiman, 2012).In the light of above given research, this paper has been proposed Modified Mean Methods for solving non-linear equations. It has been realized that the proposed Methods are performing better as compared to Bisection method.

II. PROPOSED METHODS

In this section, we have been developed several numerical iterated methods by using difference Mean Formulae, such as

Bisection Method or Arithmetic Mean:

Bisection Method somewhat called Arithmetic Mean between a, b. Bisection Method or Arithmetic Mean approximation solution `m` by the iterative scheme.

$$m = \frac{a+b}{2}$$

Similarly,

Geometric Mean

$$m = \sqrt{ab} \qquad \text{For} \quad a, b \neq 0$$

Harmonic Mean

$$m = \frac{2ab}{a+b} \qquad \text{For} \quad a, b \neq 0$$

Quadratic Mean

$$m = \sqrt{\frac{a^2+b^2}{2}}$$

Cubic mean

$$m = \sqrt[3]{\frac{a^3+b^3}{2}}$$

Heronian Mean

$$m = \frac{a+\sqrt{ab}+b}{3}$$

Now, we modified these Means by using numerical technique, such as

$$b = a + h$$

Where `h` can be written as $h = \nabla a = f(a)$, we get

$$b = a + f(a)(1)$$

Now,(1) Substitute in all the above Means, we get Modified Mean Methods, thus

Modified Bisection Method:

$$m = \frac{a+b}{2}$$

Let (a,b) be an interval, for better approximation we convert that interval into sub-interval i.e. $(a, \frac{a+b}{2})$ or $(\frac{a+b}{2}, b)$, we get

$$m = \frac{3a+b}{4}$$

or

$$m = \frac{a+3b}{4}$$

By using (1) in above, we get

$$m = a + 0.3f(a) \qquad \text{(I)}$$

$$m = a + 0.8f(a) \qquad \text{(II)}$$

If root is closer than interval point `a` then we use (I) or If root is closer than to interval point `b` then we use (II).It is Modified Bisection Method or Modified Arithmetic Mean, Likewise

Modified Geometric Mean:

By using (1) in Geometric Mean, we get

$$m = \sqrt{a(a+f(a))}$$

It is Modified Geometric Mean.

Modified Harmonic Mean:

By using (1) in Harmonic Mean, we get

$$m = \frac{2a(a+f(a))}{2a+f(a)}$$

It is Modified Harmonic Mean.

Modified Quadratic Mean:

By using (1) in Quadratic Mean, we get

$$m = \sqrt{\frac{a^2+(a+f(a))^2}{2}}$$

It is Modified Quadratic Mean.

Modified Cubic mean:

By using (1) in Cubic mean, we get

$$m = \sqrt[3]{\frac{a^3+(a+f(a))^3}{2}}$$

It is Modified Cubic Mean.

Modified Heronian Mean:

By using (1) in Heronian Mean, we get

$$m = \frac{2a+\sqrt{a(a+f(a))}+f(a)}{3}$$

It is Modified Heronian Mean.

Hence, these are Modified Mean Methods for solving nonlinear problems.

III. CONVERGENCE ANALYSIS

In this section, we have given the main results of this paper. We will give Mathematical proof that the Open Mean Methods has linear order of convergence.

Proof

By using Taylor series, firstly we are expanding $f(a)$and $[a+f(a)]$ of order of term about `$\alpha$`, we obtain

$$\text{F(a)} = e_n f`(\alpha) + o(e^2{}_n) \qquad ---(i)$$

&

$$[a + f(a)] = e_n + e_n f`(\alpha) + o(e^2{}_n) \qquad ---(ii)$$

Modified Bisection Method or Arithmetic Mean:

By using (i) in Modified Bisection Method, we gain

$$e_{n+1} = e_n + 0.3(e_n f`(\alpha) + o(e^2{}_n))$$

$$e_{n+1} = e_n[1 + 0.3f`(\alpha) + o(e^2{}_n)]$$

OR

$$e_{n+1} = e_n + 0.8(e_n f`(\alpha) + f`(\alpha) + o(e^2{}_n))$$

$$e_{n+1} = e_n[1 + 0.8f`(\alpha) + o(e^2{}_n]$$

It has shown that the Modified Bisection Method or Modified Arithmetic Mean is linear order of convergence, likewise

Modified Geometric Mean:

By using (ii) in Geometric Mean, we get

$$e_{n+1} = \sqrt{e_n(e_n + e_n f`(\alpha) + o(e^2{}_n)}$$

$$e_{n+1} = \sqrt{\mathrm{e}^2{}_n(1 + f`(\alpha) + o(e^2{}_n)}$$

$$e_{n+1} = e_n\sqrt{(1 + f`(\alpha) + o(e^2{}_n)}$$

Modified Harmonic Mean:

By using (i) and (ii) in Harmonic Mean, we get

$$e_{n+1} = \frac{2\mathrm{e}^2{}_n(1 + f`(\alpha) + o(e^2{}_n)}{e_n[2 + f`(\alpha) + o(e^2{}_n]}$$

$$e_{n+1} = e_n\frac{2(1 + f`(\alpha) + o(e^2{}_n)}{2 + f`(\alpha) + o(e^2{}_n)}$$

Modified Quadratic Mean:

By using (ii) in Quadratic Mean, we get

$$e_{n+1} = \sqrt{\frac{\mathrm{e}^2{}_n + (e_n + e_n f`(\alpha) + o(e^2{}_n))^2}{2}}$$

$$e_{n+1} = e_n\sqrt{\frac{1 + (1 + f`(\alpha) + o(e^2{}_n))^2}{2}}$$

Modified Cubic mean:

By using (ii) in Cubic mean, we get

$$e_{n+1} = \sqrt[3]{\frac{\mathrm{e}^3{}_n + (e_n + e_n f`(\alpha) + o(e^2{}_n))^3}{2}}$$

$$e_{n+1} = e_n\left(\sqrt[3]{\frac{1 + (1 + f`(\alpha) + o(e^2{}_n))^3}{2}}\right)$$

Modified Heronian Mean:

By using (i) and (ii) in Heronian Mean, we get

$$e_{n+1} = \frac{2e_n + e_n\sqrt{(1 + f`(\alpha) + o(e^2{}_n))} + e_n(f`(\alpha) + o(e^2{}_n))}{2}$$

$$e_{n+1} = e_n\frac{\left[2 + \sqrt{(1 + f`(\alpha) + o(e^2{}_n))} + (f`(\alpha) + o(e^2{}_n))\right]}{2}$$

Henceforth this has been shown that the Proposed Modified Mean Methods have linear order of convergence.

IV. RESULT AND DISSCUSSION

The developed methods have been used on few examples of non-linear functions and compared developed methods with the Bisection method in Table-1. From the numerical result of table-1, it has been observed that the Modified Mean Techniques are not only reducing iterations but also increasing accuracy perception. Mathematical package such as C++ and EXCEL have been used to justify the results and graphical representations of Modified Mean Methods, such as in Table-1

Table-1

FUNCTION	METHODS	NO OF ITERATION	Root	A E
	Bisection Method	23		$1.9209e^{-7}$
	Modified Bisection Method	7		$1.9209e^{-7}$

sinx-x+1 (1,2)	Modified Geometric Method	12	1.93456	$2.3842e^{-7}$
	Modified Harmonic Method	12		$1.9209e^{-7}$
	Modified Quadratic Method	13		$1.9209e^{-7}$
	Modified Cubic Method	12		$1.9209e^{-7}$
	Modified Heronian Method	12		$1.9209e^{-7}$
$Cosx-x^3$ (0.5,2)	Bisection Method	25	0.865474	$5.96046e^{-8}$
	Modified Bisection Method	9		$5.96046e^{-8}$
	Modified Geometric Method	22		$5.96046e^{-8}$
	Modified Harmonic Method	18		$1.78814e^{-7}$
	Modified Quadratic Method	20		$5.96046e^{-8}$
	Modified Cubic Method	23		$5.96046e^{-8}$
	Modified Heronian Method	23		$5.96046e^{-8}$
e^x-3x^2 (0.5,1)	Bisection Method	23	0.910008	$5.96046e^{-8}$
	Modified Bisection Method	8		$5.96046e^{-8}$
	Modified Geometric Method	21		$1.19209e^{-7}$
	Modified Harmonic Method	21		$1.19209e^{-7}$
	Modified Quadratic Method	21		$1.19209e^{-7}$
	Modified Cubic Method	20		$1.19209e^{-7}$
	Modified Heronian Method	20		$5.96046e^{-8}$
$e^{-x}-cosx$ (4,5)	Bisection Method	21	4.72129	$4.76837e^{-7}$
	Modified Bisection Method	9		$4.76837e^{-7}$
	Modified Geometric Method	19		$4.76837e^{-7}$
	Modified Harmonic Method	18		$4.76837e^{-7}$
	Modified Quadratic Method	19		$4.76837e^{-7}$
	Modified Cubic Method	17		$4.76837e^{-7}$
	Modified Heronian Method	18		$4.76837e^{-7}$
$4sinx-e^x$ (1,2)	Bisection Method	23	1.36496	$1.9209e^{-7}$
	Modified Bisection Method	8		$1.9209e^{-7}$
	Modified Geometric Method	25		$1.9209e^{-7}$
	Modified Harmonic Method	23		$1.9209e^{-7}$
	Modified Quadratic Method	22		$2.38419e^{7}$
	Modified Cubic Method	24		$1.9209e^{-7}$
	Modified Heronian Method	19		$1.9209e^{-7}$

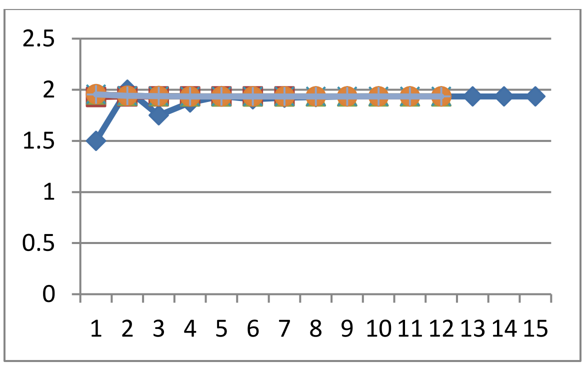

Fig.1. Comparison of accuracy analysis for the case of problem 1

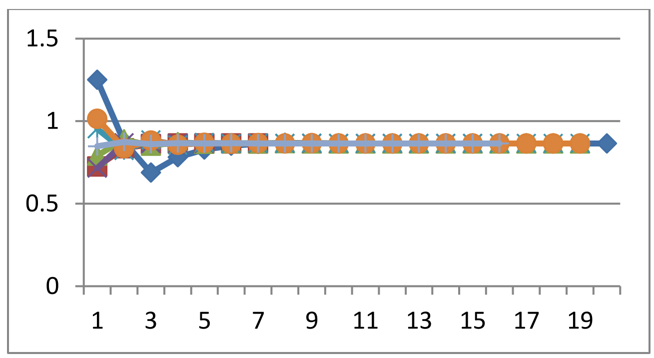

Fig.1. Comparison of accuracy analysis for the case of problem 2

V. CONCLUSION

In this paper, modified mean algorithms have been designed to estimate the root of nonlinear equations. Through the research, proposed methods are performing better than Bisection Method in terms of accuracy as well as iteration point of view. In Corollary, it has been observed from the results and comparisons that the Modified Mean Methods are providing far better results as compared to the Bisection method.

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