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Modification of Heun's Iterative Method for the Population Growth Rate Problems

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***Abstract*:** In this paper Modified Heun's algorithm of Heun's algorithm is presented with different formulations which are applied on exponential Population growth rate problems. In Heun's algorithm the average of two formulations is considered as A.M mean, where as in Modified Heun's algorithm G.M and Modified Heun's algorithm H.M are also considered as averages which are also applied on exponential population growth rate problems respectively. Comparison between numerical results of both Modified Heun's algorithm and existing Heun's algorithm shows that Modified Heun's algorithm of Heun's algorithm is more convergent then Heun's algorithm. Both algorithms will be analyzed by different errors for the convergent purpose.

Keywords: Exponential Population Growth Rate problems, Heun's Algorithm, Modified Heun's Algorithm, Convergence, Error.

I. INTRODUCTION

Differential equation arise from many problems in oscillations of mechanical and electrical system, bending of beam, conduction of heat, velocity of chemical reaction etc, and such as play a very important role in all modern and scientific and engineering studies. Differential equations whether ordinary, partial or algebraic; that evolves change of some variables with respect to other variables. Mathematical models are very useful to solve real word problems, most of differential equations are difficult to solve analytically, then it must rely some numerical method to solve them, there are number of numerical method which are used for differential equation to solve them numerically, like Euler's method Runge Kutta method Adms Bash fourth method etc. In this paper, we solve exponential population growth rate problems using Heun's algorithm and modified Heun's algorithm. There are many excellent and exhaustive text on this subject that may be consulted such as, Euler's method is presented from the point of view of Taylor's algorithm and Runge Kutta method which are used on ordinary differential equation for stability, accuracy, consistency, and convergence [1]. There methods are present to solve initial value, problems, first order Euler's, second order Heun's and rational Block method. The numerical results shows the block method is more convergent then both methods [2]. A new nonlinear adaptive numeric solution for ordinary differential equation with initial conditions the main features is to implement nonlinear polynomial expansions in a nural network-like adaptive framework [3]. There are comparative studies of numerical methods for the numerical methods namely; Runge Kutta method, Euler's method and an implicit linear multistep method of order six which are used for ordinary differential equation.

II

In these three methods the implicit linear multistep method of order six is more accurate and converges and faster than both methods [4]. Here is suggested Taylor's series expansion algorithm of numerical solution for ordinary differential equation, which are competes strongly with other existing algorithm [5]. Adams- Bashfourth method, Runge Kutta method, Adams- Moultan method which are used for ordinary differential equation and stiff problems for consistency, stability, and convergence [6]. There exist a huge number of numerical methods that iteratively construct approximation to solution of ordinary differential equation [7]. A numerical method namely, Stochastic terms involves a numeric are of pseudo random number for solving ordinary differential equations. The present work is very useful for exponential population growth rate problems and shows convergence in numerical results.

II. NUMERICAL METHOD

Numerical techniques forms an important part of solving initial and boundary value problems in ordinary differential equation; most important in case where there no closed form solution. Here is present some numerical algorithm Existing Heun's algorithm and modified Heun's algorithm of Heun's algorithm.

Population Growth Rate Differential Equation Problems. Differential equation dp/dt=kp of the Growth rate problems, p(t) be the quantity that increase with time t, in ordinary differential equation k>0, where k is proportionality constant and t is time. Then exact solution will be $p(t) = p_o e^{kt}$ Heun's algorithm for the solution of differential equation is described as $p_{n+1} = p_n + h\frac{a+b}{2} \ldots$ (1) where $a = g(t_n, p_n), b = g[t_n + h, p_n + hg(t_n, p_n)]$

Now re-write the Equation (1) by taking the functional values of a and b we got Equation (2).

$$p_{n+1} = p_n + h\frac{[g(t_n, p_n) + g(t_n + h, p_n + hg(t_n, p_n))]}{2} \ldots (2)$$

This is the Heun's algorithm for the exponential population Growth Rate problems

III. PROPOSED/ MODIFIED ALGORITHM OF HUEN'S ALGORITHM.

So, here a and b obtained from AM which is Heun's algorithm.

Here consider different Means such as G.M and H.M for modification in Huen's algorithm.

Using a and b in Equation (1) we got Equation (3) and (4)

$$p_{n+1} = p_n + h\sqrt{ab} \quad \ldots (3)$$

$$p_{n+1} = p_n + h\frac{2ab}{a+b} \quad \ldots (4)$$

Again we will substitute a and b in equation (3) and (4) respectively we get equations (5) and (6)

$$p_{n+1} = p_n + h\sqrt{g(t_n, p_n)g(t_n + h, p_n + hg(t_n, p_n))} \ldots (5)$$

$$p_{n+1} = p_n + h\frac{2[g(t_n, p_n)g[t_n + h, p_n + hg(t_n, p_n)]]}{g(t_n, p_n) + g(t_n + h, p_n + hg(t_n, p_n))} \ldots (6)$$

Equation (5) shows the modified Heun's algorithm by taking Mean HM where as equation (6) represent modified Heun's algorithm using H.M mean; now, relationship between averages in equations (2) (5) and (6) shows that A.M is greater than G.M and G.M is greater than H.M i.e A.M > G.M > H.M.

IV. NUMERICAL RESULTS

In this section, we have given some examples of exponential Population Growth Rate from open literature assess the performances of existing algorithm and modified algorithm and compare the results of Heun's algorithm with the modified Heun's algorithm and both algorithms will be analyzed by various errors. Here the main focus of this research is on increasing convergence rate of modified Heun's algorithm to Heun's algorithm.

Example 1: Consider Exponential Population Growth rate problem

$$\frac{dp}{dt} = kp, \text{ given that } \quad t \in [0.1, 1]$$

The exact solution is given by $A = 100e^{0.250679566129t}$

Table 1: Minimum Error in H.M

Time	Exact	A.M	A.E	G.M	A.E	H.M	A.E
0.1	102.538	102.632	0.094	102.629	0.091	102.626	0.088
0.2	105.141	105.334	0.193	105.327	0.186	105.321	0.180
0.3	107.810	108.106	0.296	108.097	0.287	108.087	0.277
0.4	110.547	110.952	0.405	110.939	0.392	110.926	0.379
0.5	113.353	113.872	0.519	113.855	0.502	113.839	0.486
0.6	116.230	116.869	0.639	116.849	0.619	116.828	0.598
0.7	119.181	119.945	0.764	119.921	0.740	119.897	0.716
0.8	122.206	123.103	0.897	123.074	0.868	123.045	0.839
0.9	125.308	126.343	1.035	126.31	1.002	126.277	0.969
1	128.489	129.668	1.179	129.631	1.142	129.593	1.104

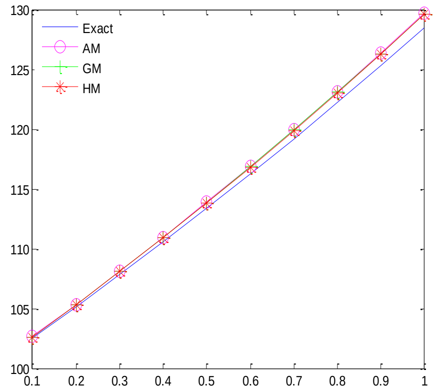

Figure 1. Exact and numerical resultsfor Example 1

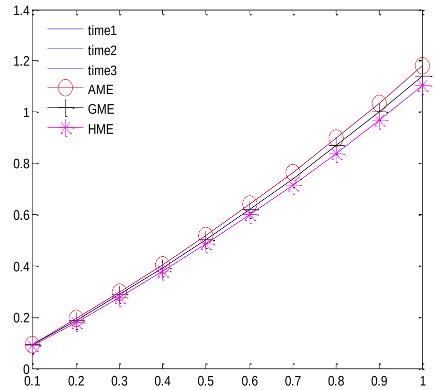

Figure 2. Errors of numerical results for Example 1

As in given Table 1. We have obtained that the minimum error of the in modified Heun's algorithm of HM mean. Modified algorithm is convergent than existing algorithm of mean AM. So, both methods are convergent, but modified algorithm is more convergent than Heun's algorithm. The error of the numerical solution gets smaller form Table 1. We find that numerical results are accurate as numerical solution is close to the exact solution. The error of modified algorithm of HM mean is less than existing algorithm. We infer that both numerical methods solve the problems quite well.

Example 2: Exponential Population Growth rate problem

$$\frac{dp}{dt} = kp, given\ that \quad t \in [0.3,3]$$

The exact solution to this problem is given by $N = 150e^{0.01335t}$

Table 2: Minimum Error in H.M

Time	Exact	AM	AE	GM	AE	HM	AE
0.3	150.601	150.632	0.031	150.630	0.029	150.620	0.019
0.6	151.206	151.266	0.06	151.265	0.059	151.263	0.057
0.9	151.813	151.903	0.09	151.901	0.088	151.899	0.086
1.2	152.422	152.543	0.121	152.54	0.118	152.537	0.115
1.5	153.034	153.185	0.151	153.181	0.147	153.178	0.144
1.8	153.648	153.83	0.182	153.826	0.178	153.821	0.173
2.1	154.264	154.478	0.214	154.473	0.209	154.468	0.204
2.4	154.883	155.128	0.245	155.123	0.24	155.117	0.234
2.7	155.505	155.782	0.277	155.775	0.27	155.768	0.263
3	156.129	156.438	0.309	156.43	0.301	156.423	0.294

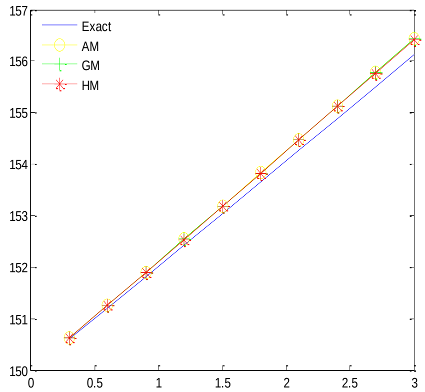

Figure 3. Exact and numerical results for Example 2

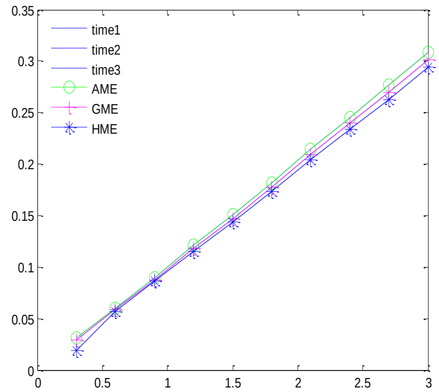

Figure 4. Errors of numerical results for Example 2

In the second example, the table 2 shows that the minimum error in the HM. Modified Heun's algorithm Existing algorithm is less convergent than modified Heun's algorithm is convergent, so modified algorithm is more convergent. The error of the numerical solution gets smaller form Table 2. The error of modified method is less than existing algorithm.

Example 3: Consider the Exponential population Growth rate problem

$$\frac{dp}{dt} = kp, given that \quad t \in [0.2, 1.8]$$

The exact solution to this problem is given by $p = 10{,}000e^{0.1t}$

Table 3: Minimum Error in H.M

Time	Exact	A.M	A.E	G.M	A.E	H.M	A.E
0.2	10202.0	10211.1	9.10	10210.9	8.9	10210.6	8.6
0.4	10408.1	10426.7	18.6	10426.2	18.1	10425.7	17.6
0.6	10618.3	10646.8	28.5	10646.0	27.7	10645.3	27.0
0.8	10832.8	10871.5	38.7	10870.5	37.7	10869.5	36.7
1.0	11051.7	11101.0	49.3	11099.7	48.0	11098.4	46.7
1.2	11274.9	11335.4	60.5	11333.8	58.9	11332.2	57.3
1.4	11502.7	11574.7	72.0	11572.8	70.1	11570.8	68.1
1.6	11735.1	11819.0	83.9	11816.8	81.7	11814.6	79.5
1.8	11972.1	12068.5	96.4	12066.0	93.9	12063.4	91.3

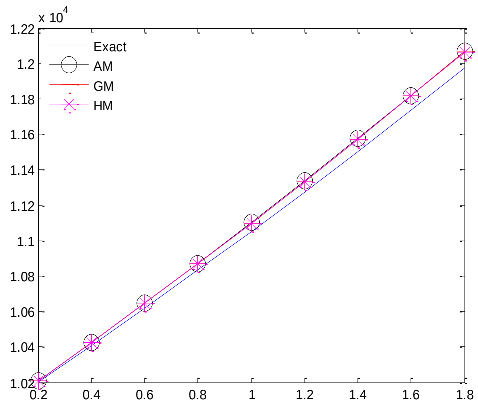

Figure 5. Exact and numerical results for Example 3

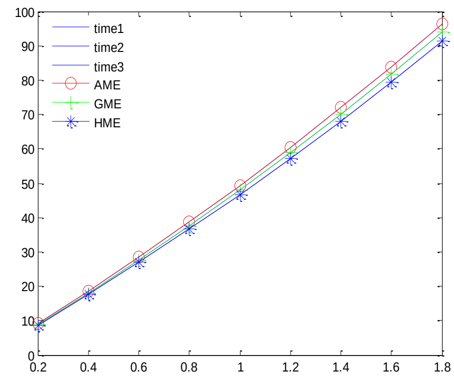

Figure 6. Error of numerical results for Example 3

As given in Table 3. obtain the minimum error of the modified Heun's algorithm, than existing method so both methods are convergent, We find that numerical results are accurate as numerical solution is close to the exact solution. The error of modified Heun's algorithm is less than existing algorithm. We find that both methods are useful for Population growth rate problems.

V. Discussion of Results

We notice that in figures the result of modified algorithm of Heun's algorithm is more convergent than existing algorithm, and also error of modified algorithm is less than existing algorithm; so that modified algorithm is more convergent.

VI. Conclusion

We have attempted exponential Population Growth rate problems using Heun's algorithm as A.M which is combination of two functions value; modified Heun's algorithm of Heun's algorithm considered as G.M and H.M; the modified Heun's algorithm is faster convergent in computation, because this algorithm is very useful for Population Growth rate problems. From numerical results we found that modified Heun's algorithms is robust than existing algorithm. That is, the modified algorithm is more efficient to solve problems including those with population growth rate.

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