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# Interpretation of Electric Charges Behaviour and Validation of Relativistic Charge Model in Special Relativity and Electromagnetism

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**Abstract:** The transformation of external electric and magnetic field from one frame of reference to another are well defined in Lorentz transformation theory and relativistic electromagnetism, but when fields produced by point charges are observed in different frames, one may not get the estimated results by applying such transformation methods as they don't replicates the relativistic charge. This paper confirms the existence of relativistic charge and investigates the behaviour of electric charges in all inertial frames by considering the charge as the relativistic quantity and explores the magnetic field created by moving charge.

In this paper, we have demonstrated and derived the relativistic charge model from the electromagnetic field transformation of individual point charges from one frame of reference to another, which will contributes to understand the differentiation of electric and magnetic fields of unit charge measured in rest and moving frames. The presented work attempts to investigate the special relativity domain that may attract the researchers to think about the peculiarities that how an electric and magnetic fields observed in rest frame would differ from moving frame. Considering charge as a relativistic quantity, some important consequences of special relativity such as charge conservation, time dilation phenomena and mass energy equivalence along with Maxwell's equation of electric flux have been derived for the confirmation of demonstrated relativistic charge equation model. The presented relativistic charge model is analyzed in the prospective of classical physics laws and derived equations and relationships confirm that the laws of physics are same in both rest and moving frame of reference.

**Keywords:** Electromagnetic Theory, Relativistic Charge, Time Dilation, Mass energy equivalence, Maxwell's equation, Coulomb's

## 1. INTRODUCTION

The modern theory of classical electromagnetism is based on theory of special relativity (Siva 2012). The dynamics of relativistic theory in any physical system greatly relies on the nature of the reference frame, either inertial or non-inertial (Wamalwa and Omolo, 2010). In Inertial frames, the laws of motion are valid, while non-inertial frames (i.e. frames of reference) are used to define physical quantities, such as position, momentum, spin, and so on (Dickson, 2004). The special relativity formulates the concepts of electric and magnetic fields under a Lorentz transformation when one considers the inertial frame of reference with respect to another (Liu, 2014). Theory of special relativity provides the basics to understand the electric and magnetic field behaviour in different frames and gives analytical and geometrical way to understand the electromagnetism in space-time symmetry (Einstein, 1952). Considering the relativistic velocity, one may also envisage that the charge and mass may not be different entities (Ghosh, *et al.,* 2013).

There has been a continuous argument from the beginning of special relativity that why the magnitude of charge remains constant when a charge particle moves with high speed as comparable to the speed of light (relativistic speed) in space under the influence of electromagnetic field, the mass turned into relativistic terminology while charge is still considered as invariant in all inertial frames (Strocchi, 2004), on the other hand, Maxwell's equations do not implicate the charge variation if charges are in moving frame (Mazauric, 2004). So consequently, this paradox is still mysterious that how a single accelerated point charge radiates electromagnetic ways. From a historical point of view, it is evident that Maxwell's equations themselves were precursors to the eventual formulation of special relativity by Albert Einstein in 1905 (Arthur, 2013). According to Purcell, it is significant to understand the effect of an electric field measured in one frame when it is stationary W.R.T a frame moving with constant velocity, then measured fields will be different in each frame (Purcell and Smith, 1986). This is imperative to consider the effect of charges in different frames to understand the relativistic electromagnetism.

In this paper, we have demonstrated the relativistic charge model which contributes to understand the differentiation of electric and magnetic field of unit charge measured in rest and moving frames. The presented work attempts to investigate the subject of special relativity in a domain that may attract the researchers to think about the peculiarities that how an electric and magnetic fields observed in rest frame would differ from moving frame. From this investigation it will be conceivable to make certain predictions about electric and magnetic fields as per

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reaction of the charges within their bounded fields. The total charge of given object is variant W.R.T Lorentz transformation when the reference frame is physically large enough to identify that charges are actually moving (Brandi, *et al.,* 1979). For example, when we heat the material the average electron velocity would increase obviously from the average nuclear velocity (Memon, *et al.*, 2013). In this work, we have considered the nucleus as our reference frame and by assuming that we are in nucleus reference frame, the charge on each individual electron increases, so we can find the difference between total charge on electrons and protons. But if we consider frame of reference outside the nucleus (i.e. that is in normal condition when we are observing the body), then we may not be able to find any difference between total charge on electrons and proton because electrons are in rest with respect to our frame of vision and we cannot observe them as moving even if we heat the material.

The rest of paper proceeds as follows. In Section 2 the relativistic charge and force transformation in rest and moving frames are discussed, while Section3 confirms the Maxwell's equation on the basis of demonstrated relativistic charge model. In Section 4 charge conservation and motion of electric charges in space time are presented. Section 5 confirms the time dilation phenomena on the basis of relativistic charge model while mass energy equivalence is demonstrated in Section 6. Section 7 discuses and confirms the derived relativistic charge model in the prospective of the classical physics laws while Section 8 conclude the work.

## 2. EXPERMENTAL ANALYSIS

### 2.1 *Relativistic Charge andForce Transformation in Rest and Moving Frames*

From beginning of modern physics, it is well established that laws of physic are valid in all inertial frames while in special relativity the mass of a charge particle varies under Lorentz transformation that can be transverse or longitudinal depending on the direction of applied force to moving particle (Calcagni, 2013). But it is still questionable that what makes electric charge to create magnetic field when it is observed in moving frame. Electric charge is considered to be invariant in all frames (Lammerzahl, *et al.*,2005), if it is the case, then one may not have the accurate information of fields in moving frame w.r.t one in rest. Some electromagnetic field transformation relations for rest and moving frames are already proved in Lorentz transformation (Nelson, 1987). By considering these arguments, this paper presents the relativistic charge that can be derived from the electromagnetic field transformation of individual point charges from one frame of reference to another.

To setup the concept, let us consider an example illustrating how transforming field's problem may be simplified into another 'frame of reference'. Let us for the moment restrict ourselves to the two parallel opposite charges $q_1$ and $q_2$along $y-axis$at rest, where $q_1$ is fixed charge and $q_2$ is pointcharge and let introduce some notations to simplify the concept, $S$and $S'$representing the rest and moving frames with a velocity $v$ in the $x$ direction respectively while $F$represents magnetic field of a charges observed by$S'$ frame. At the instant, consider that force between electric charges is only being observed in $S$frame along$y-axis$. This rest frame only observes the attractive electric force (i.e. Coulomb's force) which is equal in magnitude but opposite in direction ($F_y = -F_y = q_2E_y$) with no magnetic force as illustrated in **(Fig.1).**

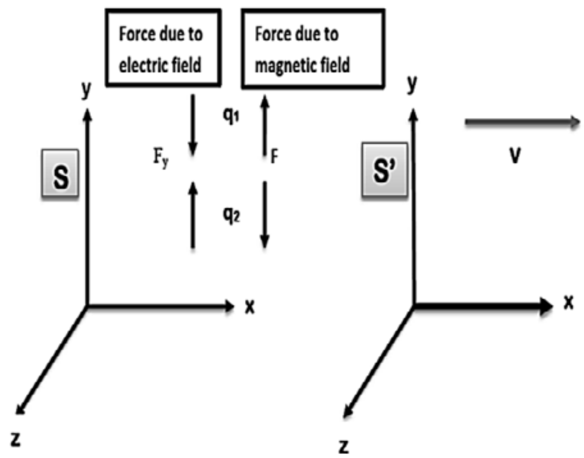


**Fig.1.Electric and magnetic force between charges being measured in two frames ($\mathbf{S}$ and $\mathbf{S}'$).**

Now consider that another frame $S'$ is moving with the velocity near to the speed of light. This frame will observe not only the electric force between two charges but also magnetic force of equal magnitude but opposite in direction ($F'_y = -F'_y = q'_2vB'$) along $y-axis$ due to the magnetic field of two charges moving relative to the $S$frame in $x-\ axis$, where $B' = \frac{\mu_0}{4\pi r^2}q'_1v$ is the magnetic field, and $\mu_0 = 4\pi\times10^{-7} \approx 1.256\ H.m^{-1} or\ N.A^{-1}$.

The electric force and net force measured by $S'$frame can be represented as: $F_y{}' = q_2E_y + q'_2vB'$and $q_2E_y = F'_y - q'_2vB'$, which are same as measured in S frame earlier. Here, negative sign $(-)$ indicates that magnetic force measured in $S'$ frame is just opposite of attractive electric force between charges with the ratio of $\frac{F_E}{F_B} = \frac{c^2}{v^2}$.

From this understanding, one may argue that magnetic field observed by $S'$ frame is actually nothing but it is relativistic effect on the charge. According to the fundamental law of physics, net electric force between charges should be same in either frame of reference. In this prospective, the above mentioned mathematical terms and notations in transformation mode from one frame to another can be represented as:

$$F_x' = F_x' = F_z' = F_z' = 0 \quad (1)$$

$$F_y{}' = F_y + q'_2 vB' = F_y + F_y' = F_y + \frac{v^3}{c^2} q'_2 E',$$

$$F_y' = F_y{}' - F_y \quad (2)$$

$$F_y = F'_y - F_y' \quad (3)$$

or

$$q_2 E = q'_2 E' - q'_2 vB' \quad (4)$$

Maxwell's equations of free space are invariant in special relativity (Cullwick, 1979), at the same time formulating the Lorentz force equation of field vectors that are transverse to the velocity $v$ become: $E' = \gamma(E + v \times B)$, where gamma $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is known as Lorentz factor.

By interpreting Lorentz force equation under the transformation of fields, Einstein didn't explain the force on electric charge moving in electromagnetic field (Janssen, 2002). At that point, Einstein was only assuming the force on a unit charge in transverse electric field $E$ (without magnetic field) equal to $E'$ or $\gamma E$, which is a divergent to the first postulate of special relativity which stares that laws of physics are same in both $S$ and $S'$ frame but cannot be linked between two frame of references, consequently the force on a moving charge in $S$ frame should be equal to electric field $E$.

To overcome this misinterpretation, we have considered the electric and magnetic fields produced by charges which led us to investigate that *"whether is it a charge that actually varies in different frames of references and ultimately causes the field of point charge to change from S to S' frame?"* By simplifying equation (4) and taking ratio of point charge $q_2$ from $S$ to $S'$ frame of references, a new relation can be developed, as:

$$\frac{q_2}{q'_2} = \frac{E' - VB'}{E} \quad (5)$$

By substituting $E'$, $E$ and $B'$ in equation (5), can be presented as:

$$\frac{q_2}{q'_2} = \frac{\frac{q'_1}{4\pi\varepsilon_0 r^2} - v^2 \frac{q'_1}{4\pi\varepsilon_0 c^2 r^2}}{\frac{q_1}{4\pi\varepsilon_0 r^2}}$$

Where $\varepsilon_0 = 8.854\ \mathrm{F.m^{-1}}$ and $c = 3 \times 10^8\ m.s^{-1}$ are permittivity of free space and speed of light in vacuum respectively. Now, solving above equation will lead as:

$$\frac{q_2}{q'_2} = \frac{q'_1\left(1 - \frac{v^2}{c^2}\right)}{q_1}$$

$$\frac{q_1 q_2}{q'_1 q'_2} = \left(1 - \frac{v^2}{c^2}\right)$$

$$q'_1 q'_2 = \frac{q_1 q_2}{\left(1 - \frac{v^2}{c^2}\right)} = \frac{q_1 q_2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)^2}} \quad (6)$$

Equation (6) shows that product of two point charges measured by $S'$ frame increases by the square of the gamma factor with respect to the product of point charges measured by $S$ frame. This actually makes electric charge to vary its electric fields from one frame to another. From this fact, we can analyse that charge actually depends upon the velocity of observing frame or charge's own velocity when it is measured in $S$ frame (Louis-Martinez, 2012) as given in the equation below.

$$q' = \frac{q}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)^2}} \quad (7)$$

Equation (7) represents relativistic charge model, where $q'$ is charge on a particle measured by moving observer with the velocity $v$ while $q$ is the charge on particle measured by observer at rest. From the basic facts, the motion of electric charges and their relativistic effects considering that total amount of force between the particles appears same in either frame of reference are same (Waterman, 1971). To confirm that whether the force measured in $S'$ frame would be identical to the force measured in $S$ frame including relativistic effect on charge, we need to find the ratio between total force in $S$ and $S'$ frame. The electric force on point charge $q_2$ along $y - axis$ in $S$ frame can therefore be represented as:

$$F_y = q_2 E \quad (8)$$

While the net force observed on point charge by $S'$ frame is given as:

$$F_y{}' = q'_2 E' - q'_2 vB' \quad (9)$$

Equation (9) represents electric force again because magnetic force is always less than electric force for the observer having velocities less than velocity of light. By taking ratio of equation (8) to (9), we get:

$$\frac{F_y}{F'_y} = \frac{\frac{q_1 q_2}{4\pi\varepsilon_0 r^2}}{\frac{q'_1 q'_2}{4\pi\varepsilon_0 r^2} - v^2 \frac{q'_1 q_2'}{4\pi\varepsilon_0 c^2 r^2}}$$

$$\frac{F_y}{F'_y} = \frac{1}{\gamma^2\left(1 - \frac{v^2}{c^2}\right)}$$

Which yields,

$$\frac{F_y}{F'_y} = 1, F_y = F'_y \quad (10)$$

Where $F_y$ and $F'_y$ represents the net forces measured in $S$ and $S'$ frames respectively.

Equation (10) confirms that the observed forces in $S$ and $S'$ frame between the charges moving parallel to each other are always identical and independent of the motion of the observer as illustrated in **(Fig.1).**

## 3. CONFIRMATION OF MAXWELL'S EQUATION USING RELATIVISTIC CHARGE MODEL:

Considering effect of special relativity only in electromagnetism, one can determine the total electric flux of charge out of the closed surface using Maxwell's point charge equation given as: $\nabla . E = \frac{\rho}{\epsilon_0}$, where "$\rho$"is charge density measured by $S$frame, which varies due to Lorentz contraction as well as relativistic charge, if the observer is in uniform motion (Turnball, 2013).So we can evaluate Maxwell's equation in $S$ frame on the basis of special relativity and relativistic charge model proposed in Fig-1. Considering equation (4), and by eliminating the relativistic charge and taking divergence of electric field, electric flux of charge in $S$ frame can be obtained as:

$$\nabla . \mathrm{E} = \lim_{\mathrm{V}\to 0} \iint \frac{\gamma \mathrm{E}'}{\mathrm{V}} \mathrm{ds}$$

$$\nabla . \mathrm{E} = \lim_{\mathrm{V}\to 0} \iint \left(\frac{\gamma \mathrm{E}'}{\mathrm{V}} - \frac{\gamma(\mathrm{v} \times \mathrm{B}')}{\mathrm{V}}\right) \mathrm{ds} \quad (11)$$

Using divergence theorem, the relationship of electric flux can be established as:

$$\nabla . E = \frac{\gamma \rho'}{\epsilon_0} - \mu_0 \gamma \rho' v^2$$

$$\nabla . E = \frac{\gamma \rho'}{\epsilon_0} - \frac{\gamma \rho' v^2}{\epsilon_0 c^2}$$

$$\nabla . E = \frac{\gamma \rho'}{\epsilon_0}\left(1 - \frac{v^2}{c^2}\right) \quad (12)$$

Since, $\rho' = \gamma\rho$, solving this we can get:

$$\nabla . E = \frac{\rho}{\epsilon_0} \quad (13)$$

Where $\rho$ is rest charge density.

Equation (13) shows that total electric flux in free space measured by observer at rest only depends on rest charge density ρ. Charge density is total charge in unit volume, length or area depending on the type (Muller, 1997). Consider that two electrons are in $S$ frame which is at rest; while another$S'$ frame is moving with the velocity comparable to the velocity of light say$V$ as shown in Fig.1. The $S'$ frame will observe the electric as well as magnetic force due to motion. The question arises that why electrons create magnetic fields (also magnetic force due to magnetic field) when they are being observed in moving $S'$ frame? According to classical theory of fields, electric charges create electric field only when they are in $S$ frame. But when another frame $S'$is moving with relativistic velocity $V$, then according to presented relativistic charge model the charge on each electron will increase as given by equation (7), so the change in electric charge of each electron causes change in electric field with respect to $S'$ frame, and according to Farady's law 'change in electric field creates magnetic field'. That's the actual reason why $S'$ frame observe the electric and magnetic forces between two charges is $S$frame. These facts revel the concept of relativistic charge which causes the electrons to create magnetic force when they are moving or being observed in moving $S'$frame.

## 4. CHARGE CONSERVATION NDMOTION OF ELECTRIC CHARGES IN SPACE TIME:

### 4.1 *Charge Conservation*

According to the charge conservation the net charge for an isolated system in the universe is constant, as the electric charge can neither be created nor destroyed. The only possibility of changing the net charge of a system strongly depends on the mechanism to bring in the charge from elsewhere or remove the charge from the system, but still the charge creation and distortion can only be happen in equal positive-negative pairs (Louis-Martinez, 2012). Most of evidences characterise that positive and negative charges are in equal quantities therefore the net charge in the universe is zero (Orito and Yoshimura, 1985). Mathematically continuity equation as shown in equation (14) well describes the constant charge of an isolated system, as:

$$Q(t'') = Q(t') + Q_{in} - Q_{out} \quad (14)$$

Where, $Q(t)$ is electric charge quantity in a specific volume at time $t$, $Q_{in}$ is the amount of charge flowing into the volume between time$t'$and$t''$, and$Q_{out}$ is the amount of charge flowing out of the volume during the same time period. Similarly, vector calculus can also be used to represent the charge conservation phenomena relating the charge density $\rho$ and electric current densityJ as shown:

$$\frac{\partial\rho}{\partial t}+\nabla.J=0 \qquad (15)$$

Or

$$\frac{\partial\rho}{\partial t}=-\nabla.J$$

Where, $\rho$ is the charge density measured in coulombs per cubic meter and $J$ is current density measured in amperes per square meter.

Equation (15) is an equivalent of four-dimensional electric current density. For the case, when we have to deal with four-current density vectors, then both special theory and proposed relativistic charge model can be taken into account.

### 4.2 *Relativistic Charge and Four-Current Density*

Relativistic charge can also be represented in terms of the Four-current density (Matveyev and Armstrong, 1966), as:

$$J^{\alpha}=\ \rho_0 U^{\alpha}=\rho\sqrt{1-\frac{v^2}{c^2}}\ \ U^{\alpha} \qquad (16)$$

Where,$\rho$ is the charge density measured by an observer at rest, and $\rho_0$ is the charge density for an observer moving at the speed $U$ along with the charges.

So realistically, the change in charge density is due to the bounded volume of charge due to Lorentz contraction as well as change in charge of electrons moving with velocity $V$ with respect to rest observer.

## 5. CONFIRMATION OF TIME DILATION USING RELATIVISTIC CHARGE MODEL:

Considering space-time symmetry with special relativity, the phenomena of time dilation can be observed in different frames (Van Holten, 1992).For the confirmation purpose of time dilation on the basis of relativistic charge model, let's us consider two electrons separated by distance$\Delta x$located along $x-axis$ as shown in **(Fig.2 (a))** where one electron is fixed along with the observer which is at rest in $S$ frame when the experiment is performed .The observer in $S$ frame can measure the coulomb's force between charges as:

$$F_x=\frac{ke^2}{\Delta x^2} \qquad (17)$$

Where $F_x$ is the Coulomb's force along $x-axis$,$k=\frac{1}{4\pi\varepsilon_0}=8.987\times10^9 Nm^2C^{-2}$ and $e=1.6\times10^{-19}C$are the charge on electrons and coulomb's constant respectively.

Now, if another observer in S′ frame moving with the velocity v, measuring the coulomb's force between two electrons as shown in **(Fig. 2(b))**, given by:

$$F'_x=\frac{ke'^2}{\Delta x'^2} \qquad (18)$$

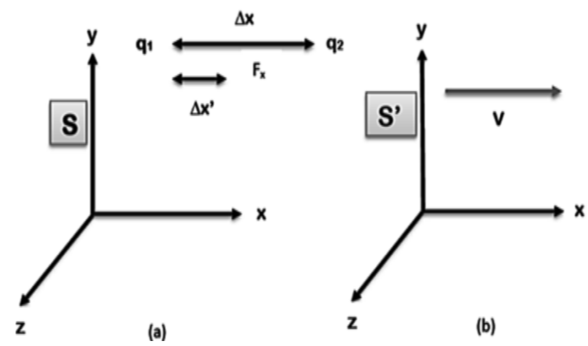


**Fig.2. Two frames S and S′ measuring coulomb's force between two rest electrons**

Assuming that the force in x direction is constant, while basic kinematics $F=ma$ can be invoked to isolate the acceleration, velocity, and position equations that can be represented as:

$$m'^{a_x}=\frac{ke'^2}{\Delta x'^2} \qquad (19)$$

$$v'^{(t)_x}=\frac{kt'^{e'^2}}{m'\Delta x'^2} \qquad (20)$$

Where $v'(t)_x$ is the velocity of electron moving through a small distance $\Delta x'_0$ measured by moving $S'$ frame, yielding:

$$\Delta x'_0=\frac{kt'^2e'^2}{m'\Delta x'^2} \qquad (21)$$

Suppose that both observers have given the identical clocks, both are witnessing the same event of an electron moving through small distance $\Delta x_0$ and $\Delta x'_0$ respectively as shown in **(Fig.3(a))** and **(Fig.3 (b)).**

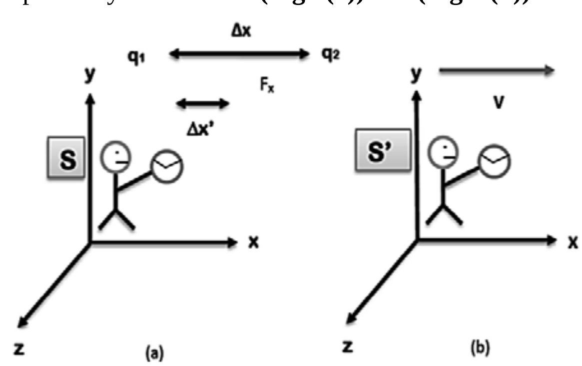


**Fig.3. Stationary and moving observers measuring the time it takes for the electron to travel in Δx the lateral direction.**

Due to relativistic effect, the witness's time by each observer will be different (Bray, 2006). According to equation (21), time measured by moving observer for the occurrence of this event will be:

$$t' = \sqrt{\frac{m'\Delta x'^2}{k\Delta x'_0 e'^2}}$$

For sake of clarity and keeping the intent of this paper, all relativistic effect will be considered in moving frame, where $m' = \gamma^3 m_0$, $e' = \gamma e_0$, $\Delta x'_0 = \gamma \Delta x_0$ and $\Delta x' = \gamma \Delta$, substituting these values in above equation, we get:

$$t' = t_0 \sqrt{\left(1 - \frac{v^2}{c^2}\right)} \qquad (22)$$

Where $t'$ is the time taken by an electron to move $\Delta x_0$ distance as measured by moving observer while $t_0$ is the time measured by rest observer. Hence, if the stationary observer analysis the moving clock, it would appear to him running slowly.

## 6. CONFIRMATION OF MASS ENERGY EQUIVALENCE ON THE BASIS OF RELATIVISTIC CHARGE MODEL:

Mass energy equivalence is the important consequence of special theory of relativity exploring the conversion of mass into energy and vice versa (Issac, Storch, Carbone, 2013).Einstein first derived this equation by considering radioactive element emitting continuous electromagnetic waves and being observed from rest and moving frames, and then he applied Maxwell's equation for further mathematical derivations. In this paper, we are assuming charge as a variable quantity which depends on the velocity of observer as given below.

$$q = \frac{q_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)^2}}$$

By squaring and cross multiplying both sides, we have:

$$q^2\left(1 - \frac{v^2}{c^2}\right) = 1$$

$$q^2c^2 - q^2v^2 = c^2 \qquad (23)$$

Taking differential of equation (21):

$$2qc^2dq - 2qv^2d\gamma - 2vq^2dv = 0$$

$$c^2dq = v^2dq + vqdv$$

$$c^2dq = v(vdq + qdv)$$

For an instance, let us restrict ourselves to charge moving in electromagnetic field. When a charge moves in electric and magnetic field, it experiences an electromagnetic force. Consider that a charge $q$ is moving with the uniformvelocity along$x - axis$with respect to rest observer in$S'$ framewhile electric and magnetic fields are applied externally along $x - axis$ and $z - axis$respectively as illustrated in **(Fig.4).**

The force experienced by charge due to magnetic field is given by:$F_B = q(v \times B)$.Where, $F_B$ is magnetic force, q is the charge moving with velocity $v$ and $B$ is applied magnetic field.Similarly, force due to electric field on a charge is given by:$F_E = qE$.Where, $F_E$ is the electric force and $E$is applied electric field.

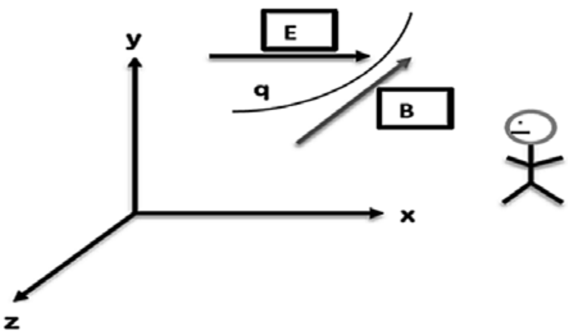


**Fig.4.Charge q moving with velocity v along x − axis in electromagnetic field**

Let us consider that electric and magnetic fields are provided in such a way that electric and magnetic force on a charge becomes equal and charge will deviate due to magnetic field and follows circular path according to Lorentz force law (Moyssides, 1989).This relationship can be expressed as:

$$F_E = F_B \qquad (25)$$

The charge experiences the centripetal force $(F_c)$due to presence of magnetic field is given as:

$$F_B = F_c = \frac{mv^2}{r}$$

$$qvB = \frac{mv^2}{r} \qquad (26)$$

Where $F_B$ or $F_c$ is the centripetal force on charge,$B$is the applied magnetic field, $m = \gamma m_0$ is transverse mass of charges because $F_c$ is perpendicular to the direction of the motion and r is the radius of circulating charges. Solving equation (26) for linear momentum of charge, we can have:

$$p = mv = qBr \qquad (27)$$

Where $p$ is linear momentum.

The work done on a charge through small distance $(ds)$can be determined by using integral as:

$k.E = \int F.ds$

According to Newton's second law of motion, the above equation becomes:

$$k.E = \int \frac{dp}{dt}.ds = \int \frac{ds}{dt} dp \quad (28)$$

Where $\frac{dp}{dt}$is rate of change of linear momentum.

$$k.E = \int v.dp \quad (29)$$

Substituting the value of $p$from equation (27) into equation (29) yields:

$$k.E = Br \int vd(q)$$

Where $B$ and $r$are the constant terms, so they can be placed out of the integral.

Since charge$q$ is variable and depends upon velocity $v$. Therefore according to rule of integration we can equate above equations as:

$$k.E = Br \int vd(q) = Br \int d(qv) \quad (30)$$

Using product rule of derivative and multiplying and dividing by$v$, we can obtain:

$$k.E = Br \int \frac{(vdq + qdv)v}{v} \quad (31)$$

Since right side of equation (24) is integral of equation (31). Hence, it can be represented as:

$$k.E = \frac{Br}{v} \int_{q_0}^{q} c^2 \, dq \quad (32)$$

Where, charge varies from $q_0$ to $q$ as it starts from rest to the velocity$v$.After solving the integration, we have:

$$K.E = \frac{Brqc^2}{v} - \frac{Brq_0c^2}{v} \quad (33)$$

Form equation (27), substituting$qBr = mv$and $q_oBr = m_ov$in equation (33) will yield,

$$K.E = \frac{mvc^2}{v} - \frac{m_o v\, c^2}{v}$$

$$K.E = mc^2 - m_oc^2 \quad (34)$$

The kinetic energy is the difference between relativistic mass energy and rest mass energy. Therefore, total relativistic energy is the sum of the kinetic energy and the rest energy as given by:

$$E = mc^2 \quad (35)$$

Equation (35) confirms the validity of proposed relativistic charge model.

## 7. DISCUSSION AND ANALYSIS OF RELATIVISTIC CHARGE MODEL:

Special relativity theory and relativistic charge should be invoked directly to verify the results of the classical forces measured in $S'$frame that must be identical to the force measured in $S$frame. Classical physics lacks the derivation of mass and charge variation; so one may not get the correct form of force equations formulized in $S$ and $S'$ frames (Jefimenko, 1999). This mass and charge variation is demonstrated by theory of special relativity (Misner and Wheeler, 1957), and is successfully proved by proposed relativistic charge model.

In this work, we have proved mass and charge variation concepts by using classical physics analysis. Initially the S frame in which both point charges are stationary is demonstrated (as viewed by rest observer)as illustrated inFig-1, at this point the only apparent field is electric, with$E = \frac{q_1}{4\pi\varepsilon_0 r^2}$.While in S′frame (as viewed by the moving observer), the point charges are in motion and thereby increasing its apparent charges from its rest value of $q_1$to $q'_1$and $q_2$to $q'_2$.

This increases both the electric and the magnetic fields of charges observed by $S'$frame. The observed force on the point charge in $S'$frame is therefore:

$$F'_y = q_2E - q_2vB = q_2E\left(1 - \frac{v^2}{c^2}\right) \quad (36)$$

Or

$$F'_y = \frac{F_y}{\gamma^2} \quad (37)$$

Equation (37) shows the difference between two versions of $F'_y$ and $F_y$which is due to the fact that the classical formulation is lacking the proper path to predict variation of electric charge in moving frame. This ultimately turns out that the first omission is offset by a second omission occurring in the classical system.

To remedy the differences between the forces observed in two frames, the relativistic charge must be considered. This decreases the force observed in $S'$ frame which in turn will give the same set of equations in either frame of references as given below.

$$F'_y = q'_2E' - q'_2vB' = \gamma^2 q_2E\left(1 - \frac{v^2}{c^2}\right) \quad (38)$$

Or

$$F'_y = q_2E \quad (39)$$

In transformation of force, equation (39) can be reproduced as:

$$F'_y = F_y \quad (40)$$

Equation (40) demonstrates that opposing magnetic force between two charges observed by $S'$ framereduces the total Relativistic force (attractive Coulomb force) which in turn becomes equal to force observed by S frame. This confirms that the force observed by S and $S'$ frames appears to be same regardless of motion, which effectively demonstrates the unrevealed part of classical physics and theory of special relativity.

## 8. CONCLUSION

From the evaluation of special relativity, electric charge being one of the fundamental quantities was considered to be constant in all frames. Special relativity theory is of a great significant to establish the concept of relativistic electromagnetism but the demonstration of electric and magnetic field transformation needs a better explanation when it subjected to different frames.

In this work, on the basis of fundamental laws we have mathematically demonstrated the charge as a variable quantity which depends on the frame velocity. Considering the charge model as a relativistic quantity, some important consequences of special relativity such as time dilation and mass energy equivalence along with Maxwell's equation of electric flux have been derived for the confirmation of proposed relativistic charge model. Considering the well demonstrated electromagnetic theory and special relativity in conjunction with the proposed relativistic charge model, this work will pave a path to validate the transformation of electric and magnetic field of a moving point charges from inertial to non-inertial frames more precisely.

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