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Numerical Results of Viscoelastic Fluid Flow in a Pipe filled with and without porous media via model of Darcy-Brinkman associated with Constitutive Oldroyd–B Model.

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Abstract: This paper presents numerical methods to solve differential equations arising from mathematical model in viscoelastic fluid flow through a circular pipe full with porous media through by using Darcy–Brinkman model includes of the conservation of momentum and conservation of mass conveyance attached with the constitutive Oldroyd–B model. The research is communicated by means of numerical results of the problems of given differential equations associated by means of initial conditions and boundary conditions, happening in the knowledge of viscoelastic flow in pipes through porous media and without porous media. Main work of this paper is signified in three mutually partial differential equations. The results are comparison to the known finite element method. Numerical calculations of the governing differential equations are accomplished accepting solver ND–Solve purpose in Mathematica.

Keywords: Viscoelastic fluid flow in circular pipe with Porous media, model of Darcy-Brinkman, Oldroyd–B constitutive model, Numerical Solutions.

1. INTRODUCTION

The learning of fluids and the forces on them is concerned with fluid Mechanics. Fluid mechanics can be separated into fluid dynamics and fluid statics. Dynamics of fluid is an vigorous field of investigation with several solved or partially unsolved problems. Dynamics of fluid can be exactly complex and it can be solved by numerical approaches bested at times, usually by the use of computers. A contemporary discipline, described CFD (computational fluid dynamics), is dedicated to this method to solving problems of fluid dynamics. Also obtaining benefit of the extremely graphic representation nature of flow of fluid is particle image of velocimetry, an investigational method for imagining and studying fluid flow dynamics. In the fluid flow dynamics, the solutions of the P, D.E. foremost the compressible flows or incompressible flows, Newtonian fluids and non-Newtonian fluids are meaningfully worried large to importance in the literature. The results of flow of Non–Newtonian fluid and Newtonian fluid in problems naturally be contingent on calculation of different fluid flow properties. Newtonian and non-Newtonian flow of fluids linked with certain important analysis are organized by technique of (Al-Fariss, and Pinder, 1987), (Abel-Malek *et al.*, 2002), (Vafai, 2002), (Kakac, *et al.*, 1991), (Rajagopal, and Gupta 1984, 1985), (Wafo 2005) and others.

The research is to arise a model that is concerning the least number of variables and parameters, and as yet comprising the facility to define the activities of viscoelastic in complex fluid flows Viscoelastic fluids have been investigated by R.G. Larson (1999) and Taha Sochi (2009, 2010) [due to their huge purposes. The nonlinear viscoelastic model is related with Oldroyd-B model and is a simplest model of second type and it appears that greatest admired in viscoelastic fluid flow modeling. Hence Viscoelastic performance will be modelled by the Oldroyd (Oldroyd-B model 1958 and Tanner/Phan-thien 1977) constitutive models in differential form and simulation developed by Phillips, van Os.(2004) (Phan-Thien .and Tanner.1977) (Rallison and Hinch 2004). (Taha 2009), (Rajagopal and Gupta 1984) (Tan and Masuoka, 2005),

Numerical methods for solving differential equations are the main research area in the computational mathematics. It becomes more and more important with the computer techniques. The research of this paper is concerned with investigative numerical results for unsteady incompressible laminar flows of viscoelastic fluid through porous media in pipes accepting the constant viscosity. The flow of viscoelastic fluid filled with porous media, supposed to be homogeneous, and isotropic can be established by using Darcy–Brinkman model comprises of the

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conservation of momentum and conservation of mass in the absence of body forces transport attached with the constitutive model of Oldroyd–B.appears that greatest admired in flow of viscoelastic fluid modeling and limitation.

This research is concerned with how to find numerical calculations of a system are determined by the use of computers adopting Mathematica solver ND-Solve. Section 2 lined with the formulations in mathematics. Section 3 related with first case or results of flow of viscoelastic fluid without porous media in pipes; results of study State problem of first case is connected in section 3.1, as numerical results of fluid flow of viscoelastic in pipes without porous media. is connected in section 3.2, Section 4 connected with results of second case of fluid flow of viscoelastic in pipes through porous media, results of study state for second case is concerned in Section 4.1.. As in section 4.2, numerical results of fluid flow of viscoelastic full with porous space in pipes. Finally the conclusions of this paper are identified in section 5.

2. FORMULATION IN MATHEMATICS

Consider in a pipe, a tubular porous layer apprehended saturated with incompressible laminar flow of viscoelastic fluid in radial route. A system of polar coordinate is applied vertically upward with radius-axis. The flow system whose main equations contains of the momentum preservation and conservation of mass transport combined with constitutive model of Oldroyd–B. The viscoelastic fluid flow through porous space is considered to have isotropic and homogeneous.

Velocity field for unidirectional flow is known as $\bar{u} = \left(u(r,t), 0, 0\right);$ wherever the velocity satisfies automatically the incompressibility fluid state. The general model of Darcy–Brinkman has been utilized for the equation of momentum and in the absence of body force; equation of continuity, general momentum equation in porous medium and equation of Oldroyd–B identifies the viscoelastic stresses in the flow of fluid in vectorial system can be agreed as below

$$\nabla . \bar{u} = 0, \quad (1)$$

$$\frac{\rho}{\varepsilon}\frac{\partial \bar{u}}{\partial t} = \frac{1}{r}\nabla([\frac{\mu_2}{\varepsilon} r \underline{\underline{d}}] + \tau) - \nabla p - \rho\,\bar{u}.\nabla\bar{u} - \frac{\mu}{K}\,\bar{u} \quad (2)$$

wherever as $\dfrac{\partial}{\partial t}$ is a temporal derivative in which t symbolizes time, $\bar{u}$ is applied for the field of velocity vector, ρ is the density of fluid and μ is whole viscosity of viscoelastic fluid individually, ∇ signifies a spatial (3-D) differential operator, μ_2 is represented for the Newtonian solvent viscosity, τ is the extra stress tensor, $\underline{\underline{d}}$ is the rate–of–strain tensor, p is the isotropic fluid pressure, ε is porosity of porous space and K is related with inherent permeability within the porous medium.
The equation related with constitutive model of Oldroyd–B describes the viscoelastic stresses in the flow of fluid can be given in the formula as below,

$$\lambda\frac{\partial \tau}{\partial t} = [2\,\mu_1\,\underline{\underline{d}}\,] - \tau - \lambda\{\,\bar{u}.\nabla\tau - \nabla\bar{u}.\tau - (\nabla\bar{u})^T.\tau\} \quad (3)$$

Hence the time in rest for viscoelastic fluid flow is specified in λ and μ_1 is connected with solute viscosity of viscoelastic fluid. As μ is used whole viscosity which is involved constant as $\mu = \mu_1 + \mu_2 = 1.$

Flow fluid of Oldroyd-B is contained on the basic equation of constitutive model, the related equations are achieved by foremost the unidirectional flow of viscoelastic fluid in pipes within porous space. The differentiation of given equations by using the transport momentum equation and Oldroyd–B constitutive equations assume that body force is absent and pressure gradient is constant, the resulting equations in the governing problem are measured for modeling in mathematics.

$$\left.\begin{aligned} &\frac{\rho}{\varepsilon}\frac{\partial u}{\partial t} = \frac{\mu_2}{\varepsilon}\frac{\partial^2 u}{\partial r^2} + \frac{\mu_2}{\varepsilon\, r}\frac{\partial u}{\partial r} + \frac{1}{\varepsilon}\frac{\partial \tau_{12}}{\partial r} + \frac{\tau_{12}}{\varepsilon\, r} - \frac{\partial p}{\partial z} - \frac{\mu}{K}u \\ &\lambda\frac{\partial \tau_{11}}{\partial t} = 2\lambda\,\tau_{12}\frac{\partial u}{\partial r} - \tau_{11} \\ &\lambda\frac{\partial \tau_{12}}{\partial t} = \mu_1\frac{\partial u}{\partial r} - \tau_{12} \end{aligned}\right\} \quad (4)$$

Where the component of velocity is indicated by u(r, t) in the axial direction and $\tau_{11}(r,t)$, $\tau_{12}(r,t)$ and $\tau_{22}(r,t)$ are used for components of the stress Tensor in axial, shear and radial direction, where second normal stress $\tau_{22} = 0$ and r is the radial direction.
The above equations system (4) can be completed dimensionless by introducing the following dimensionless variables and parameters

$$Vc\,u^* = u,\ \frac{u\,Vc}{R}\tau^* = \tau,\ R\,r^* = r,\ \frac{R}{Vc}t^* = t,$$

$$K^* = K,\ \frac{R}{Vc}\lambda^* = \lambda,\ \mu\,\mu_1^* = \mu_1 \text{ and } \mu\,\mu_2^* = \mu_2.$$

Hence u^* is dimensionless velocity and τ^* is dimensionless stress tensor, t^* is the non-dimensional time, r^* is coordinates in radial-axis, and K^* is the modified permeability for non-dimensional related with the porous space. As a radius of the pipes denoted by R and the feature velocity is concerned with Vc is applied and assumed then reference as redial velocity

$$Vc = \frac{\varepsilon R^2 \left(-\frac{\partial p}{\partial z} \right)}{\mu}$$

After dropping stars, we can write equations (4) with the non-dimensional variables and parameters as under,

$$\left.\begin{aligned} &\mathrm{Re}\,\frac{\partial u}{\partial t} = \mu_2 \frac{\partial^2 u}{\partial r^2} + \frac{\mu_2}{r}\frac{\partial u}{\partial r} + \frac{\partial \tau_{12}}{\partial r} + \frac{\tau_{12}}{r} - \frac{1}{Da}u + 1 \\ &We\,\frac{\partial \tau_{11}}{\partial t} = 2\,We\,\tau_{12}\frac{\partial u}{\partial r} - \tau_{11} \\ &We\,\frac{\partial \tau_{12}}{\partial t} = \mu_1 \frac{\partial u}{\partial r} - \tau_{12} \end{aligned}\right\} \quad (5)$$

For completing well posed problem, initial and boundary conditions are written as

$$u(t,1) = 0, \quad \text{and} \quad \frac{\partial u}{\partial t}(t,0) = 0$$

When t > 0 (6)

And initial conditions are taken as

$$u(0,r) = \tau_{11}(0,r) = \tau_{12}(0,r) = 0$$

When $0 < r < 1$ (7)

Where the Re (dimensionless Reynolds number), *we* (dimensionless Weissenberg number) and *Da* (Darcy's number for non-dimensional) are described as

$$\mathrm{Re} = \frac{R\rho Vc}{\mu},\ We = \frac{\lambda Vc}{R},\ Da = \frac{K}{\varepsilon R^2}\ \text{nd}\ \mu_1^* + \mu_2^* = 1.$$

3. Case-1: Numerical Results of viscoelastic fluid flow without porous media in pipes.

If $Da \to \infty$ (i-e, the last term of Darcy's number vanishes), So that system (5) is written en the form

$$\left.\begin{aligned} &\mathrm{Re}\,\frac{\partial u}{\partial t} = \mu_2 \frac{\partial^2 u}{\partial r^2} + \frac{\mu_2}{r}\frac{\partial u}{\partial r} + \frac{\partial \tau_{12}}{\partial r} + \frac{\tau_{12}}{r} + 1 \\ &We\,\frac{\partial \tau_{11}}{\partial t} = 2\,We\,\tau_{12}\frac{\partial u}{\partial r} - \tau_{11} \\ &We\,\frac{\partial \tau_{12}}{\partial t} = \mu_1 \frac{\partial u}{\partial r} - \tau_{12} \end{aligned}\right\} \quad (8)$$

Conditional on boundary and initial situations given are

$$u(t,1) = 0 \ \text{and} \ \frac{\partial u}{\partial r}(t,0) = 0, \text{ where } t > 0 \quad (9)$$

and initial situations are given as:

$$u(0,r) = \tau_{11}(0,r) = \tau_{12}(0,r) = 0, \text{ when } 0 < r < 1 \quad (10)$$

3.1. Study State Solution of first case

At $\frac{\partial}{\partial t} = 0$, gives the systems of equations which

$$\left.\begin{aligned} &\mu_2 \frac{\partial^2 u}{\partial r^2} + \frac{\mu_2}{r}\frac{\partial u}{\partial r} + \frac{\partial \tau_{12}}{\partial r} + \frac{\tau_{12}}{r} + 1 = 0 \\ &2We\,\tau_{12}\frac{\partial u}{\partial r} - \tau_{11} = 0 \\ &\mu_1 \frac{\partial u}{\partial r} - \tau_{12} = 0 \end{aligned}\right\} \quad (11)$$

Conditional on boundary situations are likely as $u(1) = 0$ and $\frac{\partial u}{\partial r}(0) = 0$, (12)

Steady state solution which is found numerically and the steady state solutions of (11) subject to boundary conditions (12) is resolved adopting Mathematica' solver ND Solve and figures 1–3 have been planned for some parameters and $u(r)$ at a value of range $0 \le r \le 1$ and $\tau_{11}(r)$, $\tau_{12}(r)$ at different values of μ_1.

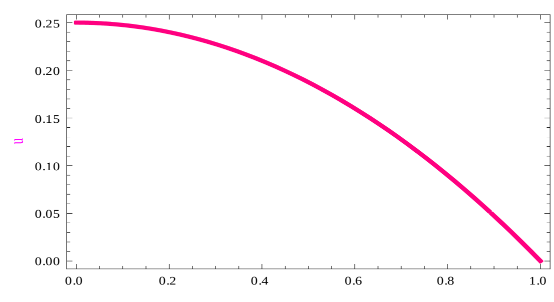
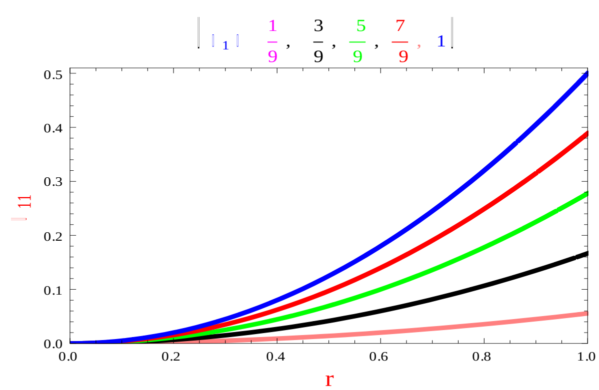


Diagram –1: solution of Steady–state of velocity u(r) of the relation (11 and 12).



Diagram–2: results of Steady–state of the component τ_{11} of normal stress (11 and 12) with

$We = 1$ and at different values of μ_1.

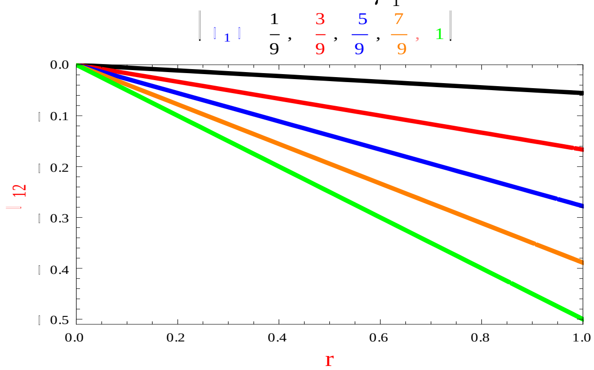


Diagram –3: Result of Steady–state of the component of shear stress τ_{12} (11 and 12) with $We = 1$ and at altered values of μ_1.

The solution of velocity u(r) is given in f diagram –1 at a range $0 \le r \le 1$. The figure –1 displays that velocity profile in the steady state takes a static curve at the given range $0 \le r \le 1$ and in velocity, there is no extra change. Results of steady–state τ_{11} and τ_{12} are conspired in the diagram 2–3 at changed values of μ_1 with $We = 1$. As the normal stress τ_{11} component is showed in figure–2 in which component of normal stress τ_{11} has small when small values of Viscoelastic solute μ_1 has taken i-e τ_{11} decreases and as component τ_{11} increases with big values of Viscoelastic solute μ_1 and has big values of solute of Viscoelastic is $\mu_1 = 1$, then maximum value of normal stress τ_{11} component is equivalent to 0.5. Whilst, shear stress τ_{12} component is showed in figure–3 and this displays that through the steady state, if the solute μ_1 decreases, then the component τ_{12} (shear stress) increases and the τ_{12} (shear stress component) reaches at its minimum value when big value of solute $\mu_1 = 1$,and minimum value of τ_{12} is - 0.5 and finally if solute μ_1 (Viscoelastic) come up to its vanishing value. then stress component is not remaining there.

3.2. Numerical Results of Flow of Viscoelastic Fluid without Porous media in Pipes

For the ruled system of PDEs (8) associated with flow of viscoelastic without porous media in pipes topic to boundary and initial situations (9 and 10), numerical result is determined adopting NDSolve of Mathematica' solver and numerical results for little variations of t (time) are expressed in the graphs 4 to 6, through growing t time.

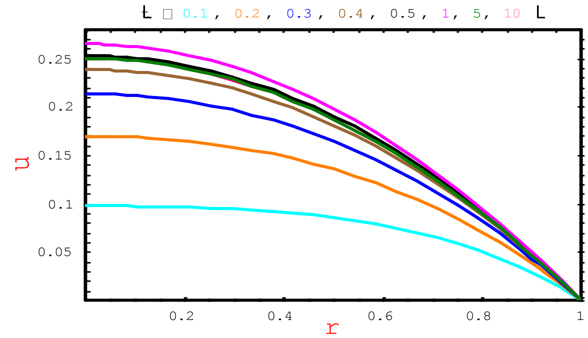
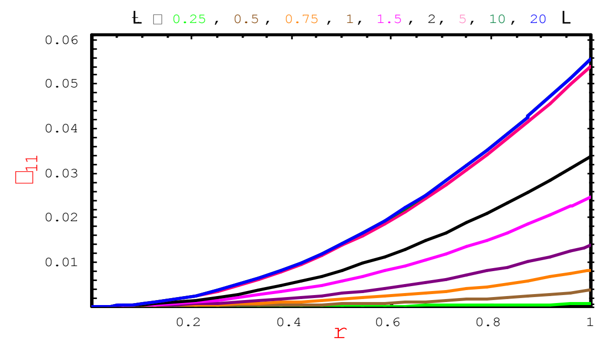
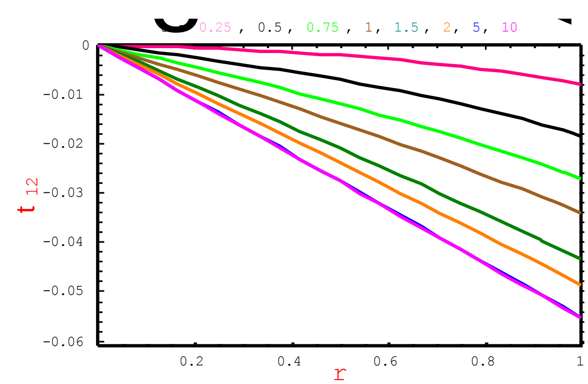


Diagram –4: Numerical results of velocity u (t, r) of the system (8) depending on boundary and initial conditions (9 and 10) through $We = 1$, $\mu_1 = 1/9$, $\mu_2 = 8/9$, and at different value of time t.



Diagram–5: Numerical results of the component τ_{11} (normal stress of the system (8) depending on conditions (9 and 10) with $\mu_1 = 1/9$, $\mu_2 = 8/9$, $We = 1$ and at different time t.



Diagram–6: Numerical results of the component τ_{12} (shear stress) of the given system (8) bound by conditions (9 and 10) with $\mu_1 = 1/9$, $\mu_2 = 8/9$, $We = 1$ and at different values of t. time

The numerical results such as u(t, r) velocity, components of $\tau_{112}(t\,.\,,r)$ normal stress and component $\tau_{12}(t\,.\,,r)$ shear stress as are existing in diagrams –4 to 6 correspondingly. diagram–4 displays, that velocity of pipe enlarges if time remains from rest and reached at maximum value of u = 0.265 and then some level reductions from the value u = 0.265 and at value of u = 0.25 than velocity got at steady–state as time $t > 5/$. Also, figure–5 show the actions of component of τ_{11} (normal stress0 enlarges relating to time t and achieved at steady–state of maximum value of $\tau_{11} = 0.056$ with respect to time $t > 7$ through analogous non–linear style. While, figure–6 show the component of τ_{12} (shear stress). From now τ_{12} clearly indicate the linear style and growths in negative direction and reached at steady–state whose value of minimum value $\tau_{12} = -0.056$ as time styles is further than five units ($t > 5$).

4. 2nd case: Explanation of fluid flow of viscoelastic in pipes by means of porous space in pipes.

The system (5) shows the PDE (partial differential equations) and lead into the fluid flow of viscoelastic using porous space in pipes accepting Oldroyd–B model revise in following equations::

$$\left.\begin{aligned} \mathrm{Re}\, u_t &= 1 + \mu_2 u_{rr} + \tfrac{\mu_2}{r} u_r + \tau_{12r} + \tfrac{\tau_{12}}{r} - \tfrac{1}{Da} u \\ We\, \tau_{11t} &= 2We\, \tau_{12} u_r - \tau_{11} \\ We\, \tau_{12t} &= \mu_1 u_r - \tau_{12} \end{aligned}\right\} \quad (13)$$

Where

$$u_t = \frac{\partial u}{\partial t},\ u_r = \frac{\partial u}{\partial r},\ u_{rr} = \frac{\partial^2 u}{\partial r^2},\ \tau_{11t} = \frac{\partial \tau_{11}}{\partial t},\ \tau_{12t} = \frac{\partial \tau_{12}}{\partial t},\ \tau_{12r} = \frac{\partial \tau_{12}}{\partial r}$$

, etc.

Depending on boundary conditions $u(t,1) = 0$ and $u_r(t,0) = 0$, when t > 0 (14)

and initial conditions are

$$u(0,r) = \tau_{11}(0,r) = \tau_{12}(0,r) = 0 \text{, when } 0 < r < 1 \quad (15)$$

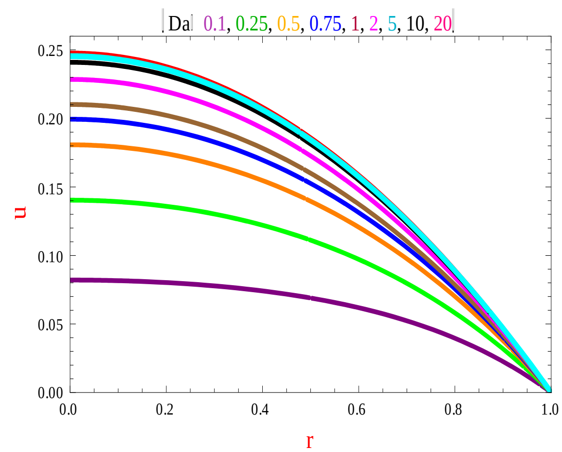
4.1. Study State Solution of flow of viscoelastic fluid using porous media in pipes.

At $\dfrac{\partial}{\partial t} = 0,$ the systems of equations (13) takes the form as under

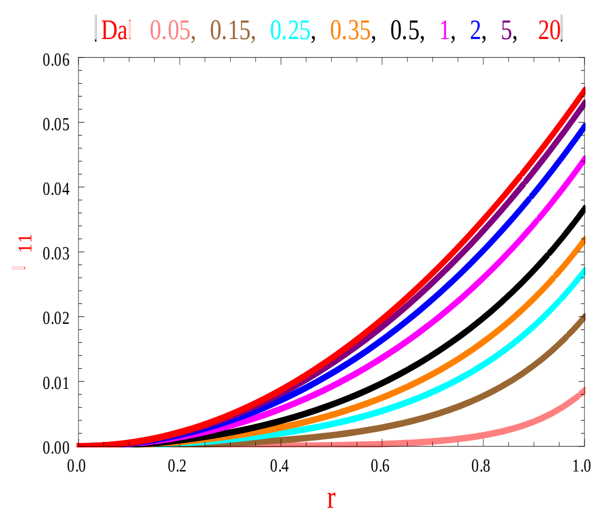
$$\left.\begin{aligned} &\mu_2 u_{rr} + \tfrac{\mu_2}{r} u_r + \tau_{12r} + \tfrac{\tau_{12}}{r} - \tfrac{1}{Da} u + 1 = 0 \\ &2We\, \tau_{12} u_r - \tau_{11} = 0 \\ &\mu_1 u_r - \tau_{12} = 0 \end{aligned}\right\} \quad (16)$$

Depending on boundary situations $u(1) = 0$ and $u_r(0) = 0$, (17)

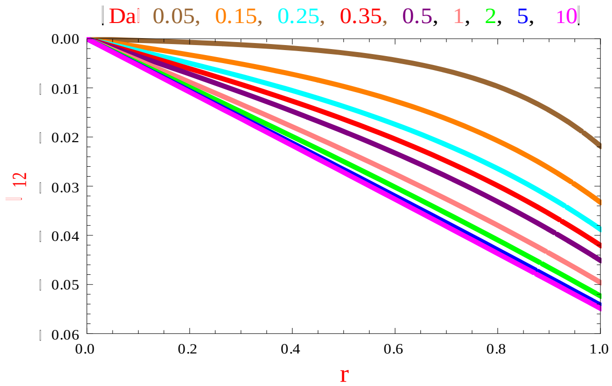
The numerically results of steady state solutions (16) subject matter to boundary conditions (17) is resolved adopting Mathematica' solver NDSolve are organised in numbers 7–9 at disparate values of *Da* (Darcy's number).



Diagram–7: Steady state solution of the velocity *u* in (16) at different values of *Da*.



Diagram–8: solution of Steady state of the component τ_{11} (normal stress) in (16) by means of $\mu_1 = \frac{1}{9}$, $We = 1$ and at disparate values of *Da*.

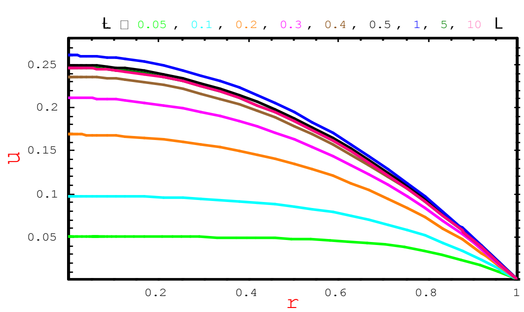


Diagram–9: result of Steady state of component τ_{12} (shear stress) in (16) with $\mu_1 = 1/9$, and at altered *Da* values.

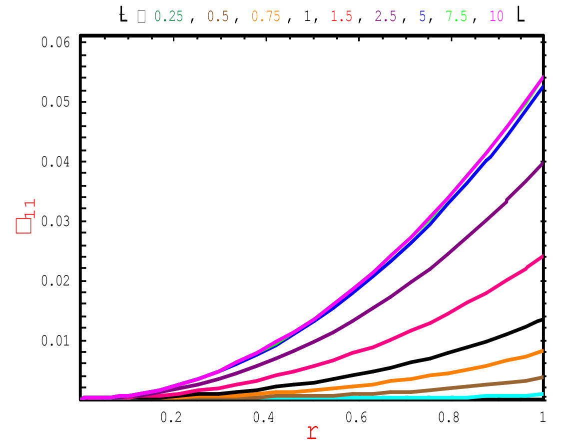
The steady state solution is relayed with velocity, normal stress component and shear stress components and plotted in graphs 7, 8 and 9 correspondingly. As the graphs 7 and 8 determine in order to the velocity u and steady component τ_{11} normal stress in height Da (Darcy's number) whilst, as Da decreases and fluid flow resistant enlarges and then velocity u and τ_{11} component decreases contained by the steady state condition and as number–9 shows to build possible in the steady state, while fluid flow in pipe comprising values of *Da* which are small, then τ_{12} component includes large values so as to if permeability decreases, then τ_{12} component increases in steady state condition and finally when Darcy's number be this close to disappearing value then there is no fluid flow in pipe.

4.2 Numerical results of fluid flow of viscoelastic concluded with porous media in pipes

The system of PD's (13) for numerical results, depending on boundary and initial conditions (14 and 15) is solved through NDSolve of Mathematica solver and results are designated in the graphs–10 to 12 by means of growing value of time.



Diagram–10: Numerical results of the velocity u of system of PDE's (1) conditional on (14–15) with $Da = 10$, $\mu_1 = 1/9$, $\mu_2 = 8/9$, $We = 1$ and at altered values of t time.



Diagram–11: Numerical solution of the normal stress component τ_{11} of the system of PDE's (13) conditional on (14–15) using $Da = 10$, $\mu_1 = 1/9$, $\mu_2 = 8/9$, $We = 1$ and at altered values t.

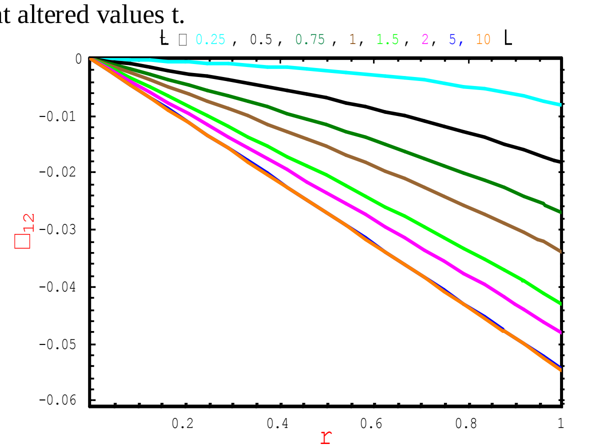


Diagram –12: Numerical results of the component τ_{12} (shear stress) of the PDE's system (13) conditional on (14–15) by way of $Da = 10$, $\mu_1 = 1/9$, $\mu_2 = 8/9$, $We = 1$ and at different time t.

In diagram–10 to 12, the numerical results of PDE's system (13 to 15) for steady parameters and at altered t time values is displayed. The results appeared graph–10, the velocity of flow concluded porous media in pipe expands as increasing of time and arrives at the value u = 0.26 as time arrive at t=1 and at that time finally velocity fluid flow purely reductions and reaches maximum value of 0.245 at steady state. As in the graph–11, the component τ_{11} shows grows with respect to t time and get up $\tau_{11} = 0.0545$ which is maximum value of τ_{11} by like non–linear style. Although, the component τ_{12} expressed in diagram –12 and determines linear tendency of growth in reverse route and achieved the minimum value of $\tau_{12} = -0.055$.

5. CONCLUSIONS

The purpose of relayed paper was to form mathematical models and to find numerical results of the problem rising in the study of flow of viscoelastic fluid in a pipe associated with and without porous media. Most vital work of this paper is represented in three jointly partial differential equations. The difficulties and complications in the problems of computational fluid mechanics, to find numerical solutions of behavior PDE's system have been determined to the investigation in this field. In our study, we worried the methods of basic equations, and texts survey with numerical results of fluid flow of viscoelastic in a pipe by using Darcy–Brinkman model coupled with Oldroyd–B constitutive model. The results of above PDE's system have solved numerically using NDSOLVE which is the important solver in Mathematica. As in the paper, firstly solutions of steady state have been established in each case of the problem linked with pipes subject to suitable boundary conditions. As well, the numerical calculations of PDE's systems problem subject to initial and boundary conditions are achieved acceptance in computer by using Solver NDSOLVE in Mathematica and explanations are designated in the figures using value of time have been increasing in each case of fluid flow in pipe through and without porous media.

Diagram of all solutions of every case were given and discussed. Numerical calculations can arrange for certain useful perceptions into the construction of results and at times may support to reach at certain results in several cases. We hope that the results may be present valuable for extra researchers in this field.

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