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Exact Solution of Non-Isothermal PTT Fluid in Post Treatment Analysis of Wire Coating with Slip Boundary Conditions

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Abstract: The problem of heat transfer analysis is considered during post treatment of wire coating process with linearly varying temperature and slip boundary conditions. The fluid is assumed to be viscoelastic obeying the non-linear rheological constitutive equation proposed by Phan-Thien & Tanner and Phan-Thien. This constitutive model simulates many polymer melts and solutions therefore, the present study is useful in a number of practical situations. For sake of simplicity, the problem is formulated in dimensionless form with the help of non-dimensional parameters. The explicit expressions for velocity, average velocity, volume flow rate and temperature distribution are derived. This appears to be the first study of the post treatment problem with a viscoelastic fluid. The scope of the present study is quite wide.

Keywords: Wire Coating; PTT Fluid; Linearly Varying Temperature; Slip Boundary Conditions.

1. INTRODUCTION

Fluids used for manufacturing purposes in chemical industry are mostly classified as non-Newtonian fluids. Generally, the non-Newtonian fluids are complex mixtures such as slurries, plastics, pastes, gels, polymer solutions etc (Harris, 1977), (Rajagopal, 1982), (Erdogan, 1981). Phan-Thien Tanner (PTT) fluids fall in the class of non-Newtonian fluids due to their rheological equation of shear stress. The PTT is simple quasi-linear viscoelastic model, derived by (Phan-Thien, 1977), (Phan-Thien, 1978). This model incorporates shear thinning, shear viscosity, normal stress differences and the elongation behavior of the fluids. The PTT model is increasingly applied to predict the flow and heat transfer of viscoelastic fluids. Recently, a sequence of papers has appeared which present the analytical results of a PTT fluids model for channel flows (Oliveira, 1999) (Cruz, *et. al*, 2005).

The wire coating is an important and oldest process using an extruder, dating back to the 1840s (Tadmor, 1979). Polymer extrudate is used to coat a wire for the purpose of mechanical strength and environmental protection. Many applications exist of wire coating in engineering disciplines such as chemical and industrial engineering. Therefore some researchers motivated to investigate the wire coating process (Denn, 1980) (Basu, 1981) and give some theoretical, numerical and experimental results. The treatment of coated wire after leaving the die is a posttreatment problem. The wire after leaving the die depends on the quality of the material used for coating and the temperature. The posttreatment of wire coating was studied by Kasajima and Katsuhiko Ito (Kasajima, 1973). They consider that the polymer obey the power law fluid model. Moreover, they examine the problem with no slip conditions and constant temperature at boundaries. There efforts establish the expression for velocity and temperature distribution. The objective of the present paper is to examine the laminar flow of viscoelastic fluids obeying the PTT fluid model and investigate the heat transfer with slip boundary conditions and linearly varying temperature.

2. BASIC EQUATIONS

The basic equations governing the flow of an incompressible fluid with thermal effects are:

$$\nabla \cdot \underline{u} = 0, \tag{1}$$

$$\rho \frac{D\underline{u}}{Dt} = div\underline{T} + \rho \underline{f}, \tag{2}$$

$$\rho c_p \frac{D\Theta}{Dt} = k_T \nabla^2 \Theta + \Phi, \tag{3}$$

where $\underline{u}$ is the velocity vector, ρ is the constant mass density, $\underline{f}$ is the body force, D/Dt denotes the material derivative, Θ is the fluid temperature, k_T is the thermal conductivity, c_p is the specific heat, $\underline{L}$ is the gradient of velocity vector $\underline{u}$, Φ is the dissipation function and $\underline{T}$ is the Cauchy stress tensor defined as

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$$\underline{T} = -p\underline{I} + \underline{S}. \quad \textbf{(4)}$$

In Eq. (4) p is the pressure, $\underline{I}$ is the identity tensor and $\underline{S}$ is the extra stress tensor.

The model adopted here to accommodate the viscoelastic behavior of the fluid is PTT model may be expressed as

$$\Psi(tr\underline{S})\underline{S} + \lambda \overset{\nabla}{\underline{S}} = 2\eta \underline{A}_1 \quad \textbf{with} \quad \Psi(tr\underline{S}) = 1 + \frac{\varepsilon\ \lambda}{\eta}(tr\underline{S}). \quad \textbf{(5)}$$

Here η is the constant viscosity coefficient of the fluid, λ is the relaxation time, $tr\underline{S}$ is the trace of the stress tensor $\underline{S}$ and $\underline{A}_1$ the deformation rate tensor defined by

$$\underline{A}_1 = \underline{L}^T + \underline{L}, \quad \textbf{(6)}$$

where T denote the transpose of matrix.

In Eq. (5) $\Psi(tr\underline{S})$ is the stress function in which ε is related to the elongation behavior of the fluid. For $\varepsilon = 0$, the model given in Eq. (5) reduces to the well-known Max-well model.
The upper contra-variant convected derivative designed by ∇ over $\underline{S}$ in Eq. (5) is defined as

$$\overset{\nabla}{\underline{S}} = \frac{D\underline{S}}{Dt} - \left[(\nabla\underline{u})^T \underline{S} + \underline{S}(\nabla\underline{u})\right]. \quad \textbf{(7)}$$

3. FORMULATION AND SOLUTION OF THE PROBLEM

The geometry under consideration is shown systematically in **(Fig. 1)**, where the polymer extrudate is denoted by the solid line. For investigation of flow behavior of a polymer used in wire coating, we divide the flow transversely into countless short sections as shown in broken lines in **(Fig.1)** and assume that that each section has the same shape. Therefore, we analyze only one section shown in **(Fig.2)**.

For appropriate investigation we choose cylindrical coordinates (r,θ,z), for the stated problem such that r is perpendicular to the direction of flow.

Consider the flow an incompressible PTT fluid of constant density in posttreatment of wire coating. The wire at temperature Θ_1 and of radius kR_0 (where R_0 is the radius of coated wire and k is the dimensionless ratio of radii, such that, $0 < k < 1$) is dragged in the z direction through a PTT fluid polymer (II) with a velocity V_1 and the gas (III) close to the polymer (II) is at temperature Θ_2 and flowing with a velocity V_2. We consider slippage exists at the contact surfaces of wire, polymer, and the gas as shown in **(Fig.2)**.

Assuming that the flow is steady, laminar, unidirectional and axisymmetric:

We seek a velocity field of the form

$$u = v = 0, \quad w(r), \quad \textbf{(8)}$$

$$\underline{S} = \underline{S}(r) \quad \textbf{and} \quad \Theta = \Theta(r). \quad \textbf{(9)}$$

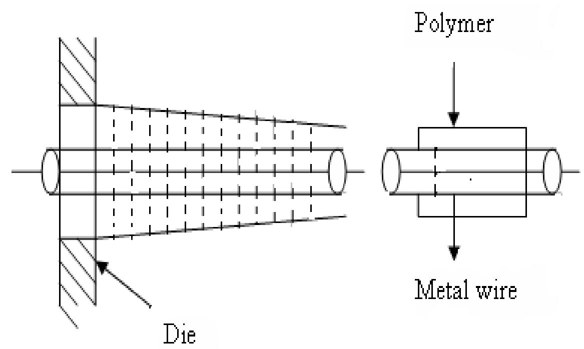

Fig-1 Schematic profile of polymer extrudate in wire coating.

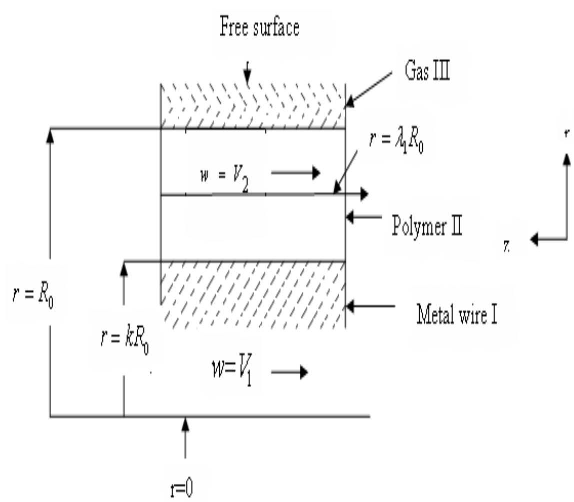

Fig-2 Drag flow in wire coating.

Using Eq. (8) the continuity equation (1) is satisfied identically. Inserting Eqs. (8) and (9) in (5-7) we obtain the following non-zero components of the stress tensor $\underline{S}$ and the dissipation function Φ are:

$$S_{zz} = 2\frac{\lambda}{\eta} S_{rz}^2 \quad , \quad \textbf{(10)}$$

$$S_{rz} + 2\varepsilon\left(\frac{\lambda}{\eta}\right)^2 S_{rz}^3 = \eta\frac{dw}{dr}, \quad \textbf{(11)}$$

$$\Phi = S_{rz}\frac{dw}{dr}. \quad \textbf{(12)}$$

Similarly, with the help of velocity field and non-zero components of the stress tensor $\underline{S}$ the momentum equation in the absence of body forces reduces to

$$\frac{\partial p}{\partial r} = 0 \quad , \qquad (13)$$

$$\frac{\partial p}{\partial \theta} = 0 , \qquad (14)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{d}{dr}\left(rS_{rz}\right). \qquad (15)$$

In view, of Eqs. (13) and (14) it is concluded that p is a function of z only.

Consider the axial pressure gradient $-\partial p/\partial z = \Omega$, where Ω is constant.

Integrating Eq. (15) we get

$$S_{rz} = -\frac{\Omega}{2}r + \frac{C_1}{r}, \quad \textbf{where } C_1 \textbf{ is an arbitrary constant of integration.} \qquad (16)$$

The boundary conditions are:

$$w = V_1 + \gamma \left\lfloor S_{rz} \right\rfloor_{r=kR_0} \quad \text{at } r = kR_0 ,$$

$$w = V_2 - \gamma \left\lfloor S_{rz} \right\rfloor_{r=R_0} \quad \text{at } r = R_0 , \qquad (17)$$

in which γ is a slip parameter.

Depending on the location of the maximum fluid velocity that may exists in the region (II) in Fig 2, the shear rate γ_0 may be written as

$$\gamma_0 = -\frac{dw}{dr} , \quad \text{for } kR_0 \le r \le \lambda_1 R_0 , \qquad (18)$$

$$\gamma_0 = \frac{dw}{dr} , \quad \text{for } \lambda_1 R_0 \le r \le R_0 , \qquad (19)$$

in which λ_1 is the dimensionless radial position where the maximum velocity occurs [25].

In other words, at $r = \lambda_1 R_0$, the velocity of polymer become maximum and we have

$$\frac{dw}{dr} = 0 , \quad \text{at } r = \lambda_1 R_0 , \qquad (20)$$

and therefore

$$S_{rz} = 0, \quad \text{at } r = \lambda_1 R_0 , \qquad (21)$$

Now solving Eq. (16) combine with (21) we obtain the integration constant existing in Eq. (16) as

$$C_1 = \frac{1}{2}\lambda_1^2 R_0^2 \Omega . \qquad (22)$$

The explicit expression for shear stress is given as

$$S_{rz} = \begin{cases} \dfrac{\Omega}{2}\left(\dfrac{\lambda_1^2 R_0^2 - r^2}{r}\right) & \text{for } kR_0 \le r \le \lambda_1 R_0 \\ \dfrac{\Omega}{2}\left(\dfrac{r^2 - \lambda_1^2 R_0^2}{r}\right) & \text{for } \lambda_1 R_0 \le r \le R_0 \end{cases} \qquad (23)$$

On inserting, Eq. (23) into (11) we obtain first order system of two ordinary differential equations defined for velocity field in two different domains as

$$\eta\frac{dw}{dr} = \frac{\Omega}{2}\left(\frac{\lambda_1^2 R_0^2 - r^2}{r}\right) + 2\varepsilon\left(\frac{\lambda}{\eta}\right)^2\left(\frac{\Omega}{2}\left(\frac{\lambda_1^2 R_0^2 - r^2}{r}\right)\right)^3 \quad \text{for } kR_0 \le r \le \lambda_1 R_0 \qquad (24)$$

$$\eta\frac{dw}{dr} = \frac{\Omega}{2}\left(\frac{r^2 - \lambda_1^2 R_0^2}{r}\right) + 2\varepsilon\left(\frac{\lambda}{\eta}\right)^2\left(\frac{\Omega}{2}\left(\frac{r^2 - \lambda_1^2 R_0^2}{r}\right)\right)^3 \quad \text{for } \lambda_1 R_0 \le r \le R_0 \qquad (25)$$

with the boundary conditions

$$w = V_1 + \frac{\gamma G}{k}\left(1 - k_1^2\right) \quad \text{at } r = kR_0 ,$$

$$w = V_2 + \gamma G\left(1 - \sigma^2\right) \quad \text{at } r = R_0 , \qquad (26)$$

where G, k_1 and σ are constants given as

$$G = \frac{1}{2}\Omega\lambda_1^2 R_0^2 , \; k_1 = \frac{k}{\lambda_1}, \; \text{and} \quad \sigma = \frac{1}{\lambda_1}$$

Assuming the temperature of the boundaries are linearly varying with z according to the expressions

$$\Theta(kR_0, z) = \Gamma z + F(kR_0) = \Theta_1, \text{ and } \Theta(R_0, z) = \Gamma z + F(R_0) = \Theta_2 , \qquad (27)$$

we set

$$\Theta(r, z) = \Gamma z + F(r) , \qquad (28)$$

where Γ is the temperature gradient.

Using Eqs. (12), (23) and (28), the energy equation (3) reduces to system of ordinary differential equations due to different shear stress in the region (II) given as:

$$k_T\left(\frac{d^2F}{dr^2}+\frac{1}{r}\frac{dF}{dr}\right)+\frac{\Omega}{2}\left(\frac{\lambda_1^2R_0^2-r^2}{r}\right)\frac{dw}{dr}-\rho c_p w\Gamma=0, \qquad \text{for } kR_0\le r\le \lambda_1 R_0, \tag{29}$$

$$k_T\left(\frac{d^2F}{dr^2}+\frac{1}{r}\frac{dF}{dr}\right)+\frac{\Omega}{2}\left(\frac{r^2-\lambda_1^2R_0^2}{r}\right)\frac{dw}{dr}-\rho c_p w\Gamma=0, \qquad \text{for } \lambda_1R_0\le r\le R_0, \tag{30}$$

At this stage, it is convenient to introducing the following non-dimensional parameters

$$r^*=\frac{r}{R_0},\ w^*=\frac{w}{V_2},\ \gamma^*=\frac{\gamma}{R_0},\ U=\frac{V_1}{V_2},$$

$$\wedge_1=\frac{\Omega R_0^{\,2}V_1}{2\eta V_2},\ \wedge_2=\frac{\varepsilon\Omega^3R_0^{\,4}\lambda^2}{4\eta^3V_2},\ \wedge_3=\frac{\Omega R_0^{\,2}}{2\eta V_2},$$

$$\wedge_4=\frac{\rho\ \ c_p\Gamma R_0^2}{\eta V_2},$$

$$\Theta^d=\begin{cases}\Theta_0^d\left(\dfrac{F(r)-F(kR_0)}{F(\lambda_1R_0)-F(kR_0)}\right), & \text{for } r\le\lambda_1R_0\\[2ex] \Theta_0^d\left(\dfrac{F(r)-F(R_0)}{F(\lambda_1R_0)-F(R_0)}\right), & \text{for } r\ge\lambda_1R_0\end{cases}$$

where Θ^d is the dimensionless temperature and Θ_0^d are defined as follows

$$\Theta_0^d=\begin{cases}k_T\left\{\dfrac{F(\lambda_1R_0)-F(kR_0)}{\mu\ \ V_2^2}\right\}, & \text{for } r\le\lambda_1R_0\\[2ex] k_T\left\{\dfrac{F(\lambda_1R_0)-F(R_0)}{\mu\ \ V_2^2}\right\}, & \text{for } r\ge\lambda_1R_0\end{cases}$$

The system of Eqs. (24-26) and (28), (29) with (27), after dropping the "$*$" take the following form:

$$\frac{dw}{dr}=\Lambda_1\left(\frac{\lambda_1^2-r^2}{r}\right)+\Lambda_2\left(\frac{\lambda_1^2-r^2}{r}\right)^3, \qquad \text{for } k\le r\le\lambda_1, \tag{31}$$

$$\frac{dw}{dr}=\Lambda_1\left(\frac{r^2-\lambda_1^2}{r}\right)+\Lambda_2\left(\frac{r^2-\lambda_1^2}{r}\right)^3, \qquad \text{for } \lambda_1\le r\le1, \tag{32}$$

with boundary conditions

$$w=U+\frac{\gamma\ \ G}{k}\left(1-k_1^2\right),\ \text{at } r=k, \tag{33}$$

$$w=1+\gamma\ \ G\ \left(1-\sigma^2\right),\quad \text{at } r=1, \tag{34}$$

and the energy equation becomes

$$\frac{d^2\Theta^d}{dr^2}+\frac{1}{r}\frac{d\Theta^d}{dr}+\Lambda_3\left(\frac{\lambda_1^2-r^2}{r}\right)\frac{dw}{dr}-\Lambda_4w=0, \qquad \text{for } k\le r\le\lambda_1, \tag{35}$$

$$\frac{d^2\Theta^d}{dr^2}+\frac{1}{r}\frac{d\Theta^d}{dr}+\Lambda_3\left(\frac{r^2-\lambda_1^2}{r}\right)\frac{dw}{dr}-\Lambda_4w=0, \qquad \text{for } \lambda_1\le r\le1, \tag{36}$$

with boundary conditions

$$\left.\begin{array}{l}\Theta^d(k)=0,\ \Theta^d(\lambda_1)=\Theta_0^d,\ \text{for}\ \ k\le r\le\lambda_1\\ \text{and}\\ \Theta^d(\lambda_1)=\Theta_0^d,\ \ \Theta^d(1)=0,\ \text{for}\ \lambda_1\le r\le1\end{array}\right\} \tag{37}$$

The boundary conditions given in Eq. (33) and (34) are appropriate for solutions of Eq. (31) and (32) respectively. We solve accordingly, and obtain the following expression for velocity fields

$$\begin{aligned}w(r)=\frac{1}{4k^2r^2}\Big[&\left(2\wedge_1k^2r^2-6\wedge_2k^2r^2\lambda_1^2-2\wedge_2\lambda_1^6\right)\left(k^2-r^2\right)+\wedge_2k^2r^2\left(k^4-r^4\right)\\&+\left(12\wedge_2k^2r^2\lambda_1^2-4\wedge_1k^2r^2\lambda_1^2\right)(\ln k-\ln r)+4Gkr^2\gamma\left(1-k_1^2\right)+4k^2r^2U\Big]\end{aligned} \tag{38}$$

for $k\le r\le\lambda_1$

and

$$\begin{aligned}w(r)=\frac{1}{4r^2}\Big[&\left(6\wedge_2r^2\lambda_1^2-2\wedge_1r^2+2\wedge_2\lambda_1^6\right)\left(1-r^2\right)-\wedge_2r^2\left(1-r^4\right)\\&-4r^2\lambda_1^2\ln r\left(\wedge_1-3\wedge_2\lambda_1^2\right)+4Gkr^2\gamma\left(1-\sigma^2\right)+4r^2\Big]\qquad \text{for } \lambda_1\le r\le1\end{aligned} \tag{39}$$

The expressions in equation (38) and (39) are the velocity fields in the combined drag and pressure-driven flow. Here, the values of λ_1 depend on the wire speed and pressure drop, whereas values of λ_1 in the pressure driven flow are obtained by setting the wire speed equal to zero.

At $r = \lambda_1$, it follows that from equation (38) and (39) that

$$\frac{1}{k^2}\left[\left(2\wedge_1 k^2 r^2 - 6\wedge_2 k^2 r^2 \lambda_1^2 - 2\wedge_2 \lambda_1^6\right)\left(k^2 - r^2\right) + \wedge_2 k^2 r^2\left(k^4 - r^4\right) + \left(12\wedge_2 k^2 r^2 \lambda_1^2 - 4\wedge_1 k^2 r^2 \lambda_1^2\right)\left(\ln k - \ln r\right) + 4G\ k\ r^2 \gamma\left(1 - k_1^2\right) + 4k^2 r^2 U\right]$$
$$= \left[\left(6\wedge_2 r^2 \lambda_1^2 - 2\wedge_1 r^2 + 2\wedge_2 \lambda_1^6\right)\left(1 - r^2\right) - \wedge_2 r^2\left(1 - r^4\right) - 4r^2 \lambda_1^2 \ln r\left(\wedge_1 - 3\wedge_2 \lambda_1^2\right) + 4G\ k\ r^2 \gamma\left(1 - \sigma^2\right) + 4r^2\right]. \quad \textbf{(40)}$$

The expression in equation (40) is used for determining the values of λ_1, which depends on the velocity ratio U, dimensionless wire radius k, slip parameter γ and the dimensionless parameters $\wedge_1$, $\wedge_2$, $\wedge_3$, $\wedge_4$. The detail theoretical and experimental analysis is given in (Han, 1978) for this value. For the determination of λ_1, one must resort to a trial and error procedure, using some kind of numerical schemes.

The average velocity is

$$w_{ave} = \frac{2}{1-k^2}\int_k^1 w\, r\, dr \quad \textbf{(41)}$$

At the cross-section, within the die, the volume flow rate is

$$Q = \int_k^1 2\pi\, r\, w(r) dr \quad \textbf{(42)}$$

As the velocity field is continuous on the value λ_1, so the volume flow rate can be re-written as

$$w_{ave} = \frac{2}{1-k^2}\left(\int_k^{\lambda_1} r\, w(r) dr + \int_{\lambda_1}^1 r\, w(r) dr\right) \quad \textbf{(43)}$$

Use of equation (39) and (40) in equation (43), we obtain

$$\begin{aligned}
w_{ave} = \frac{1/2k^2}{\left(1-k^2\right)}\Big(\Big(&\Lambda_1 k^4 - 3\Lambda_2 \lambda_1^2 k^4 + \Lambda_2 \lambda_1^6 + 6\Lambda_2 \lambda_1^2 k^2 \ln k - 2\Lambda_1 \lambda_1^2 k^2 \ln k - 3\Lambda_2 \lambda_1^2 k^2 \\
&+ \Lambda_1 \lambda_1^2 k^2 + 2Gk\gamma\left(1-k_1^2\right) + 2k^2 U\Big)\left(\lambda_1^2 - k^2\right) + \left(-\frac{1}{2}\Lambda_1 k^2 + \frac{3}{2}\Lambda_2 k^2 \lambda_1^2\right)\left(\lambda_1^4 - k^4\right) \\
&- \frac{1}{6}\Lambda_2 k^2\left(\lambda_1^6 - k^6\right) - 2\Lambda_2 \lambda_1^6 k^2\left(\ln\lambda_1 - \ln k\right) + 6\Lambda_2 \lambda_1^6 k^2\left(\lambda_1^2 \ln\lambda_1 - k^2 \ln k\right) \\
&+ \left(3\Lambda_2 \lambda_1^2 - \Lambda_1 - \Lambda_2 \lambda_1^6 + \frac{1}{2}\Lambda_2 + \lambda_1^2\left(\Lambda_1 - 3\Lambda_2 \lambda_1^2\right) - 2Gk\gamma\left(1-\sigma^2\right) + 2\right)\left(1-\lambda_1^2\right) \\
&+ \frac{1}{2}\left(\Lambda_1 - \Lambda_2\right)\left(1-\lambda_1^4\right) + \frac{1}{6}\Lambda_2\left(1-\lambda_1^6\right) - 2\Lambda_2 \lambda_1^6 k^2 \ln\lambda_1 + 2\lambda_1^4\left(\Lambda_1 - 3\Lambda_2 \lambda_1^2\right)\ln\lambda_1\Big).
\end{aligned} \quad \textbf{(44)}$$

Similarly, the volume flow rate is obtained from equation (42) and is given as

$$\begin{aligned}
Q = \frac{\pi}{2k^2}\Big(\Big(&\Lambda_1 k^4 - 3\Lambda_2 \lambda_1^2 k^4 + \Lambda_2 \lambda_1^6 + 6\Lambda_2 \lambda_1^2 k^2 \ln k - 2\Lambda_1 \lambda_1^2 k^2 \ln k - 3\Lambda_2 \lambda_1^2 k^2 \\
&+ \Lambda_1 \lambda_1^2 k^2 + 2Gk\gamma\left(1-k_1^2\right) + 2k^2 U\Big)\left(\lambda_1^2 - k^2\right) + \left(-\frac{1}{2}\Lambda_1 k^2 + \frac{3}{2}\Lambda_2 k^2 \lambda_1^2\right)\left(\lambda_1^4 - k^4\right) \\
&- \frac{1}{6}\Lambda_2 k^2\left(\lambda_1^6 - k^6\right) - 2\Lambda_2 \lambda_1^6 k^2\left(\ln\lambda_1 - \ln k\right) + 6\Lambda_2 \lambda_1^6 k^2\left(\lambda_1^2 \ln\lambda_1 - k^2 \ln k\right) \\
&+ \left(3\Lambda_2 \lambda_1^2 - \Lambda_1 - \Lambda_2 \lambda_1^6 + \frac{1}{2}\Lambda_2 + \lambda_1^2\left(\Lambda_1 - 3\Lambda_2 \lambda_1^2\right) - 2Gk\gamma\left(1-\sigma^2\right) + 2\right)\left(1-\lambda_1^2\right) \\
&+ \frac{1}{2}\left(\Lambda_1 - \Lambda_2\right)\left(1-\lambda_1^4\right) + \frac{1}{6}\Lambda_2\left(1-\lambda_1^6\right) - 2\Lambda_2 \lambda_1^6 k^2 \ln\lambda_1 + 2\lambda_1^4\left(\Lambda_1 - 3\Lambda_2 \lambda_1^2\right)\ln\lambda_1\Big).
\end{aligned} \quad \textbf{(45)}$$

The expression for temperature distribution in the domain $k \le r \le \lambda_1$ and $\lambda_1 \le r \le 1$ can be obtained from Eqs. (35-37), with the help of velocity distributions given in Eq. (38) and (39) as follows

$$\Theta^{d}(r)=$$

$$\frac{1}{288k^2r^2\left(\ln k-\ln\lambda_1\right)}$$

$$\Big[\Big(\Big(\Big(576\wedge_2 k^2r^2\lambda_1^6\wedge_3-144\wedge_1 k^2r^2\lambda_1^4\wedge_3+72\wedge_2 k^2r^2\lambda_1^6\wedge_4\Big)\ln r\ln k+126\wedge_1 k^2r^2\lambda_1^4\wedge_3$$
$$-440\wedge_2 k^2r^2\lambda_1^6\wedge_3+205\ \wedge_2 k^2r^2\lambda_1^6\wedge_4-81\wedge_1 k^2r^2\lambda_1^4\wedge_4-288k^2r^2\,\Theta_0^d\Big)\left(\ln r-\ln k\right)$$
$$+\Big(72\wedge_2\ \ k^2r^2\lambda_1^6\wedge_3\ln r\ln k+18\wedge_1 k^6r^2\wedge_3\ \ +8\wedge_2 k^8r^2\wedge_3-576\wedge_2 k^2r^2\lambda_1^6\wedge_3\ln r\ln k$$
$$+144\wedge_1 k^2r^2\lambda_1^4\wedge_3\ln r\ln k\ \ -16\wedge_2 k^8r^2\wedge_4\ \ +81\wedge_2 k^6r^2\lambda_1^2\wedge_4\Big)\ \left(\ln r-\ln\lambda_1\right)-\left(72\wedge_2\ \ k^2r^2\right.$$
$$\lambda_1^6\wedge_4\ln\lambda_1\ln k+18\wedge_1 k^2r^6\wedge_3-8\wedge_2 k^2r^8\wedge_3-576\wedge_2 k^2r^2\lambda_1^6\wedge_3\ln\lambda_1\ln k+9\wedge_1 k^2r^6\wedge_4$$
$$\left.+144\wedge_1 k^2r^2\lambda_1^4\wedge_3\ln\lambda_1\ln k-27\wedge_2 k^2r^6\lambda_1^2\wedge_4+2\wedge_2 k^2r^8\wedge_4\right)\ \left(\ln k-\ln\lambda_1\right)\ -\Big(144\wedge_1 k^2r^2\lambda_1^2\wedge_3$$
$$-432\wedge_2 k^2r^2\lambda_1^4\wedge_3-72\wedge_1 k^2r^2\lambda_1^2\wedge_4+180\wedge_2 k^2r^2\,\lambda_1^4\wedge_4\Big)\ \left(k^2\ln r-r^2\ln k\right)\ \ +\ \Big(36\wedge_1\ \ k^4r^2\wedge_4$$
$$-108\wedge_2 k^4r^2\lambda_1^2\wedge_4+18\wedge_2 k^6r^2\wedge_4\ \ \left(\lambda^2\ln r-r^2\ln\lambda_1\right)+72k\ \ r^2\wedge_4\left(k^2\ln\lambda_1+r^2\left(\ln k-\ln\lambda_1\right)\right.$$
$$+72k\ \ r^2\wedge_4\left(k^2\ln\lambda_1+r^2\left(\ln k-\ln\lambda_1\right)+\lambda_1^2\left(\ln r-\ln k\right)\right)\left(k\ \ U+G\gamma\right)\ \ -\ln\lambda_1\ \ \Big(216\wedge_2 k^2r^2\lambda_1^4\wedge_4^2$$
$$+144\wedge_1 k^2r^2\lambda_1^2\wedge_3\left(r^2-k^2\right)\ +72\wedge_2\lambda_1^8\wedge_3+144\wedge_1 k^2r^2\lambda_1^2\wedge_3\Big)\ln\lambda_1\left(r^2-k^2\right)\ +72\wedge_2\lambda_1^8\wedge_3$$
$$+144\wedge_1 k^2r^2\lambda_1^2\wedge_3\Big)\ln\lambda_1\left(r^2-k^2\right)\ +\Big(216\wedge_2 k^2r^2\lambda_1^4\wedge_4\ln\lambda_1\ \ \left(\ln r-\ln k\right)\ \ -216\wedge_2 k^2r^2\lambda_1^4\wedge_4$$
$$\ln k\ \ \left(\ln r-\ln k\right)+72\wedge_1 k^2r^2\lambda_1^2\wedge_3\ln k\ln\lambda_1-72\wedge_1 k^2r^2\lambda_1^2\wedge_3\ln r\ln\lambda_1+36\wedge_1 k^4r^2\wedge_3\ln k$$
$$+18\wedge_2 k^2r^2\lambda_1^4\wedge_3\ln k+36\wedge_2 r^2\lambda_1^6\wedge_3\ln k-72\wedge_1 k^2r^2\lambda_1^2\wedge_3\left(\ln k\right)^2\ \ -108\wedge_2 k^4r^2\lambda_1^2\wedge_3\ln k$$
$$+72\wedge_1 k^2r^2\lambda_1^2\wedge_3\ln r\ln k+36\wedge_2 k^2r^2\lambda_1^4\wedge_3\ln k\Big)\left(r^2-\lambda_1^2\right)\ +\Big(144\ \ G\ \ k\ \ r^2\gamma\wedge_4 k_1^2\ln r$$
$$-72\ \ G\ \ k\ \ r^2\gamma\wedge_4\ln r\ \ \left(k^2-\lambda_1^2\right)+72\ \ \wedge_2 k^2r^2\lambda_1^2\wedge_3\ln k\ \ \left(k^4-r^4\right)-36\ \ \wedge_2 r^2\lambda_1^4\wedge_3\ln r\ \ \left(k^4-\lambda_1^4\right)$$
$$+72\ \ \wedge_2 k^2r^2\lambda_1^2\wedge_3\ln k\left(r^6-\lambda^6\right)-72\wedge_2 r^2\lambda_1^2\wedge_3\ln r\ \ \left(k^6-\lambda_1^6\right)\ \Big]$$

$$\text{for}\quad k\le r\le\lambda_1 \qquad (46)$$

where

$$\Theta^{d}(r)=$$

$$\frac{1}{288k^2r^2\ln k}\Big[\Big(144\wedge_1 r^2\lambda_1^2\wedge_3-18\wedge_1 r^2\wedge_3-8\wedge_2 r^2\wedge_3+576\wedge_2 r^2\lambda_1^6\wedge_3\ln r\ln\lambda_1$$
$$-72\wedge_2 r^2\lambda_1^8\wedge_3-144\wedge_1 r^2\lambda_1^4\wedge_3\ln r\ln\lambda_1-27\wedge_1 r^2\wedge_4+72\wedge_2 r^2\lambda_1^6\wedge_4\ln r\ln\lambda_1$$
$$-16\wedge_2 r^2\wedge_4+72r^2\wedge_3+72Gr^2\gamma\wedge_4-72Gr^2\gamma\sigma^2\wedge_4-36\wedge_2 r^2\lambda_1^6\wedge_4+81\wedge_2 r^2\lambda_1^2\wedge_4$$
$$-216\wedge_2 r^2\lambda_1^4\wedge_4+72\wedge_1 r^2\lambda_1^2\wedge_4\Big)\left(\ln r-\ln\lambda_1\right)+\Big(432\wedge_2 r^2\lambda_1^4\wedge_3-126\wedge_1 r^2\lambda_1^2\wedge_3$$
$$+27\wedge_1 r^2\wedge_4+18\wedge_2 r^2\lambda_1^2\wedge_4-72r^2\wedge_3-72Gr^2\gamma\wedge_4+72Gr^2\gamma\sigma^2\wedge_4+36\wedge_2 r^2\lambda_1^6\wedge_4$$
$$-81\wedge_2 r^2\lambda_1^2\wedge_4+216\wedge_2 r^2\lambda_1^2\wedge_4-27\wedge_2 r^2\lambda_1^2\wedge_4-72\wedge_1 r^2\lambda_1^2\wedge_4+9\wedge_1 r^2\wedge_4\Big)$$
$$\left(\lambda^2\ln r-r^2\ln\lambda_1\right)+\left(27\wedge_2 r^2\lambda_1^2\wedge_4-9\wedge_1 r^2\wedge_4\right)\ \left(\lambda^4\ln r-r^4\ln\lambda_1\right)+\Big(8\wedge_2 r^2\wedge_3$$
$$-2\wedge_2 r^2\wedge_4\Big)\ \ \left(\lambda^6\ln r-r^6\ln\lambda_1\right)+\left(216\wedge_2 r^2\,\lambda_1^4\wedge_4\ln r\ln\lambda_1\ \ -18\wedge_1 r^4\wedge_3\ln\lambda_1\ \right)\left(r^2-\lambda_1^2\right)$$
$$+72\wedge_2\lambda_1^2\wedge_3\ln\lambda_1\left(r^6-\lambda_1^6\right)+288\ \ r^2\Theta_0^d\ \ \ln r\Big]$$

$$\text{for}\quad \lambda_1\le r\le 1 \qquad (47)$$

The temperature distribution for equal temperature at the boundaries of domain can be obtained by substituting $\Theta_0^d = 0$ in Eqs. (46) and (47) respectively.

4. CONCLUSION

Exact solutions have been derived for the nonlinear viscoelastic PTT fluids in posttreatment analysis of wire coating. These flows are formed by the combination of an imposed constant pressure gradient in the axial/longitudinal direction and the movement of a wire in that direction. In this study, the solutions of the Navier-Stokes and energy equation are established for the radial variation of the velocity and temperature. Due to positive and negative shear stress a value λ_1 is defined, where the rate of shear stress is zero i.e., velocity is highest. The location of the maximum fluid velocity moves between the wire and gas at the surface of coated wire, which depends on the drag of wire and pressure gradient. This appears to be the first study of the posttreatment problem with a viscoelastic fluid. The scope of the present study was quite wide and will be fruitful for mathematicians, engineers and scientist for the future work.

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