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Numerical Study of Stresses of Rotating Mixing Flow within a Container

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Abstract: In the present study, the numerical solution of stresses are analysed of Bird Carreau model of two–dimensional rotating flow. The problem is relevant to the food industry. The solutions are obtained through a finite element semi–implicit Taylor–Galerkin/pressure–correction scheme. The method is based on second order multi–stage time–stepping algorithm posed in a cylindrical polar coordinate system. The predicted results are validated against previously simulated results and are observed identical with one and another.

Keywords: Rotational Mixing, Contra Rotating Stirrer, Bird Carreau Model

1 INTRODUCTION

This study is the continuation of our previous published work. (Memon, et *al.*, 2011) The main focus of this analysis is to be capable to design a machine for mixing of industrial importance. The core attention is to maximize the rate of work done and reduce the power consumption on the dough kneading, using computational study of industrial related problem. A more informed understanding of the processing problem is in the report of Biniding *et al.*, 2002. The geometry considered in this case is consisting cylindrical tub along with a pair of rotating rods (stirrers) which are attached eccentrically with lid of the container. Both stirring rods are rotating in opposite direction rotational direction of outer tub (Memon, 2013).

The computational geometry and considered fluid are investigated in current analysis which appears the intermediate step in the development of dough kneading mixer using numerical simulation of such types of flows (Baloch *et al.*, 2003). In the industries, the mixers domain partly fills, that is driven within the geometry with a couple of stirrers which rotates eccentrically about the axis of outer container (Biniding *et al.*, 2002). In this paper, the dynamics of such rotating mixing flows have been numerically analysed, which is an important phase of chain of rotating mixing flows. The inertial effects, impact of rotational velocity and influence dimensionless parameter for Bird–Carreau model fluid are comprehensively analysed.

The semi implicit Taylor–Galerkin/Pressure–Correction (TGPC) time marching scheme adopted (Bochkarev and Matveenko, 2013). The two dimensional incompressible flow is modelled via so called TGPC finite element scheme posed in cylindrical polar coordinate system which applies a temporal discretisation in a Taylor series prior to a Galerkin spatial discretisation. A semi implicit treatment for diffusion is employed to address linear stability constraints. An inelastic model with shear rate dependent viscosity is considered (Sujatha *et al.*, 2002).

In this study, the solutions of field of interest are depending upon the rotational velocities, various inertial levels and dimensionless parameter (which is fixed at 0.6). The velocity gradient ($\nabla \boldsymbol{v}$), shear rate ($\dot{\gamma}$) and shear stress ($\tau_{r\theta}$) for Bird–Carreau model fluids are considered to express the use of a numerical flow solver as predictive tool for dough kneading (Baloch and Webster, 2003).

2 MATERIAL AND METHOD

Problem Specification

The cylindrical tub with a couple of stirring rods is considered. The fluid is driven by the outer container wall and two rotating cylindrical rods fixed eccentrically at the head of the tub by a lid. Contra rotating rods with angular speed angular speed half, same and double (u_θ = 0.5, 1.0 and 2.0) of stirrers against the speed of outer tub which is 1.0 (Memon *et al.* 2011) are analysed. For the study of computational

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domain, the finite element mesh with triangular elements is employed.

For the detail initial and boundary conditions the readers are advised to see Memon, (2013).

Governing System of Equations

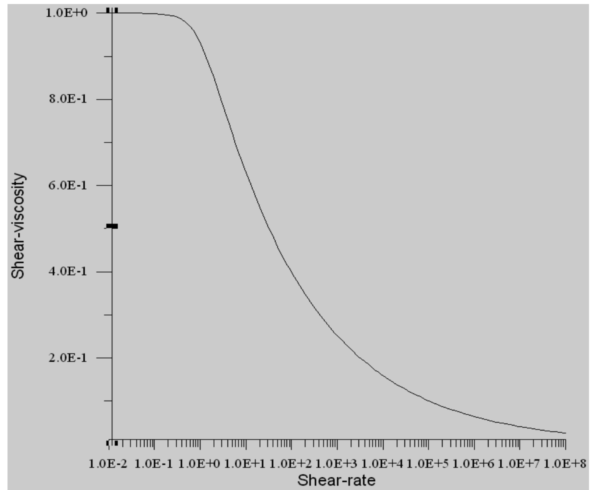
Two-dimensional system of cylindrical coordinate is employed over the domain Ω. In the absence of the body forces, the system of equations is adopted through the conservation of mass and the conservation of momentum transport equations as $\nabla\cdot\boldsymbol{u} = 0$ and $\rho\frac{\partial \boldsymbol{u}}{\partial t} = \nabla.\boldsymbol{\sigma} - \rho\boldsymbol{u}.\nabla\boldsymbol{u}$ respectively. Applying the finite element approximation, the multi stages fully discrete system adopted (for details of terms and matrices of the numerical scheme, readers are referred Memon, 2013). To quantify viscous behavior as a function of shear rate, a Bird–Carreau model for viscosity μ is employed to reflect shear thinning properties for a generalized Newtonian or inelastic fluid T is

$$T = 2\mu(\dot{\gamma})\boldsymbol{d}$$

where $\mu(\dot{\gamma})$ is known as the viscosity function which is dependent upon the shear rate $(\dot{\gamma})$. For the Bird–Carreau model, the viscosity demonstration takes the form

$$\mu(\dot{\gamma}) = \mu_\infty + (\mu_0 - \mu_\infty)[1 + (\lambda\dot{\gamma})^2]^{\frac{(n-1)}{2}}$$

where μ_0, μ_∞, λ and n are the zero shear rate, infinite shear rate limit viscosity, material constant and dimensionless parameter taken as 1 kg/m^{-s}, 0.01, 1 and 0.6 respectively.



Variation of viscosity with shear rate according to the Bird–Carreau model

3. NUMERICAL RESULTS AND DISCUSSIONS

Numerical solutions of the rotational velocities (0.5, 1.0 and 2.0) of stirrers with respect to various inertial levels (0.08, 0.8 and 8.0) are investigated. The predicted solutions of all fields of interest are displayed through contour plots. These are plotted from non–dimensional minimum value (marked by oval shape) to non–dimensional maximum value (marked by square shape), over a range for inertial level from Re = 0.08, 0.8 and 8.0 in **(Fig. 1, 2 and 3)** respectively.

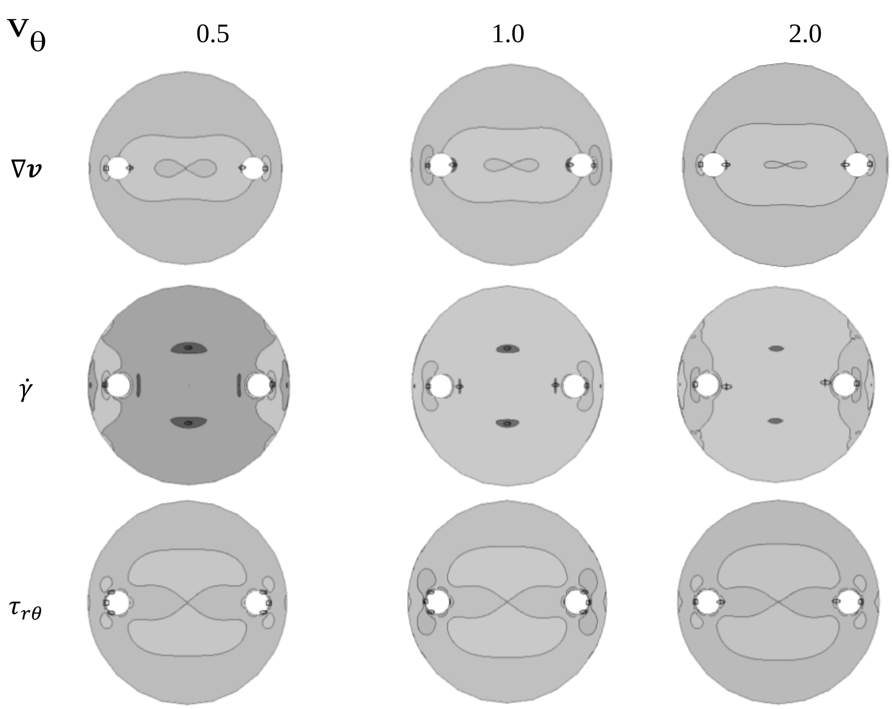


Fig. 1: Impact of rational velocity of stirrers on velocity gradient, shear rate, shear stress of Bird–Carreau model fluid for inertial level 0.08 and dimensionless parameter is 0.6.

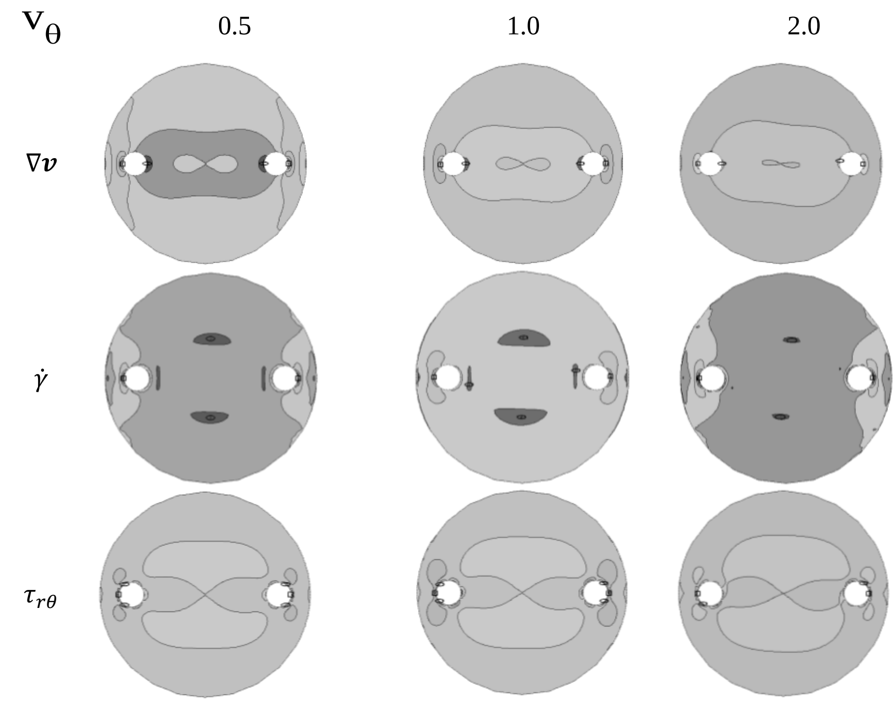


Fig. 2: Impact of rational velocity of stirrers on velocity gradient, shear-rate, shear-stress of Bird–Carreau model fluid for inertial level 0.8 and dimensionless parameter is 0.6.

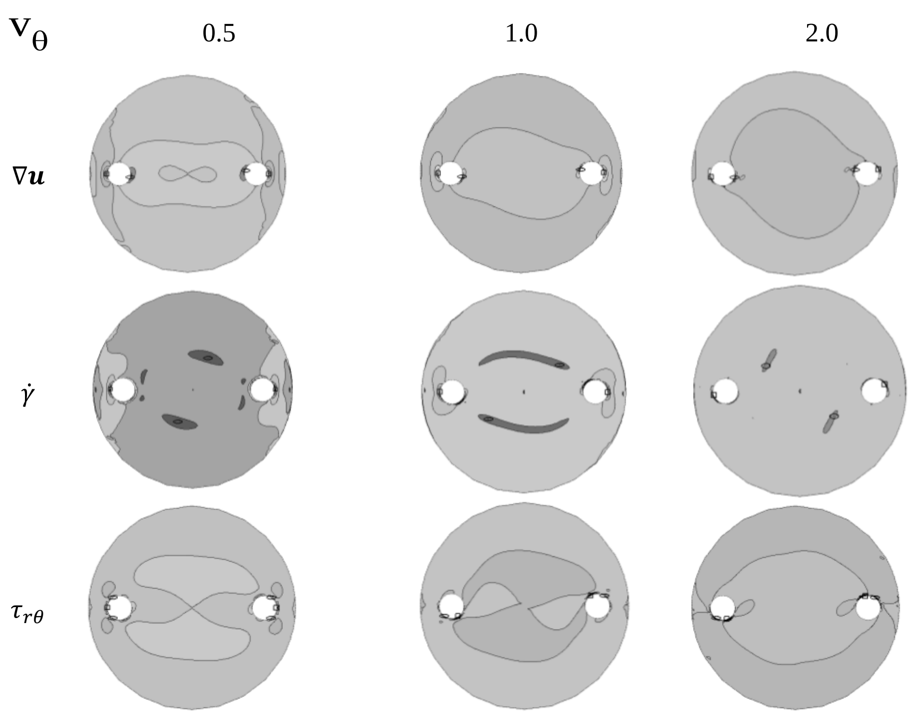


Fig. 3: Impact of rational velocity of stirrers on velocity gradient, shear-rate, shear-stress of Bird–Carreau model fluid for inertial level 8.0 and dimensionless parameter is 0.6.

Effects of increasing inertia on velocity gradient, shear-rate and shear stress: The Reynolds number 0.08 for contra-rotating stirrers, equivalent field kinematic data in all variables are shown in figure 1. Various velocities for Bird Carreau model fluids, with particular reference to localized velocity gradient, shear–rate and shear stress are discussed. At inertial level 0.08, no symmetry break–up is observed. The non-dimensional maxima in all fields of interest appeared at lip of the stirrer in gap between outer wall and both stirrers. The non-dimensional minima is noticed at opposite lip of the stirrers in the case of velocity gradient for all three cases of velocities. The minimum value of shear rate is noticed in upper and lower region of the computational domain at half speed, as speed increased, this value changes the position and appeared near the stirrers in the wide gap between both stirrers. In the case of shear stress, the minimum value appeared at upper and lower lip of the stirrers at half and same speed but it seemed at lip of the stirrers on the horizontal axis line.

As inertia increases, i.e. 0.8 and 8.0, the non dimensional maxima and minima remain at same place at all three values of velocity but a noticeable feature emerged, that is, contours of velocity gradient are showing stretching behavior in opposite rotational direction of stirrers as inertia increases as well as velocity increases. In the case of shear rate, the maxima appeared at lip of stirrers in the narrow gap and minima in the upper and lower region. The contours are elongating in the rotational direction outer container as velocity increases which replicate the real manner of the original problem. The maximum value of shear stress at both value of inertia at lip of stirrers and minimum on upper and lower lip at entry and exit at inertial level 0.8 and half speed but when inertia and speed of stirrers increases, the minimum value of shear stress appeared at lip of stirrers just after the releasing of the fluid from narrow gap and maximum value just after this place.

The minima and maxima of Computational results of velocity gradient (∇v), shear rate ($\dot{\gamma}$) and shear stress ($\tau_{r\theta}$) for Bird–Carreau model fluids are tabulated in the (**Table-1**) For the velocity gradient, minimum value decreases as inertia and the speed of stirrers upto almost double, increase in the maxima is observed upto five times with respect to increase in speed. In the case of shear rate and shear stress, at low inertial value, increase is almost double however, at maximum value of inertia, is approximately double to four times increase is noted. The numerical solutions and available experimental data are found with good agreement. The convergence and stability of the adopted algorithm using two dimensional contra rotating flows is proved from the computed results for the full incompressible Navier–Stokes equations.

Table-1: Computational results of velocity gradient (∇v), shear rate ($\dot{\gamma}$) and shear stress ($\tau_{r\theta}$) for Bird–Carreau model fluids.

Re		0.08		0.8		8.0	
Variables	Speed of Stirrer	Min	Max	Min	Max	Min	Max
∇u	0.5	-2.868	8.675	-2.866	8.677	-2.850	8.853
	1.0	-4.656	12.30	-4.663	12.30	-7.859	12.70
	2.0	-8.339	19.99	-8.452	20.04	-17.66	40.28
$\dot{\gamma}$	0.5	0.002	8.812	0.008	8.814	0.008	8.990
	1.0	0.003	12.57	0.009	12.57	0.006	12.97
	2.0	0.02	20.55	0.01	20.59	0.007	44.73
$\tau_{r\theta}$	0.5	-1.810	4.337	-1.819	4.338	-1.913	4.426
	1.0	-2.558	6.150	-2.680	6.148	-4.295	13.62
	2.0	-4.169	9.999	-4.743	10.02	-23.52	30.79

4. CONCLUSION

The stresses are analysed within the cylindrical domain with a pair of contra rotating stirrers. The effects of inertia, velocity and non-dimensional parameter on the velocity gradient, shear arte and shear stress have investigated. The attractive features emerges into view are; more shearing rate and homogenised mixing of the Bird–Carreau model fluid occurs in the closed region between stirrers and outer container, and the maximum rate of work done approach at same place. This is fully replicating the nature of the physical industrial problem.

REFERENCES:

Binding D. M., M. A. Couch K. S., Sujatha, M. F. Webster (2002)‘ Experimental and numerical simulation of dough kneading in filled geometries’, J. Foods Eng., 57, 1–13.

Baloch, A., and M. F. Webster, (2003) ‘Distributed Parallel Computation for Complex Rotating Flows of Non-Newtonian Fluids’, Int. J. Num. Meth. Fluids, 43, 1301-1328.

Bochkarev S. A., and V. P. Matveenko. (2013) ‘Numerical Analysis of Stability of a Stationary or Rotating Circular Cylindrical Shell Containing Axially Flowing and Rotating Fluid’, International Journal of Mechanical Sciences, 68, 258-269.

Memon, R. A., M. A. Solangi, and A. Baloch, (2011) ‘Analysis of Stresses and Power Consumption of Mixing Flow in Cylindrical Container’, Punjab U. J. of Mathematics, 43, 47–67.

Memon. R. A., (2013) ‘Finite Element Modeling of Complex Rotational Flows’, PhD thesis, Centre for Advanced Studies in Pure and Applied Mathematics, Bahauddin Zakariya University Multan, Pakistan,.

Solangi, M. A., A. Baloch and R. A. Memon, (2011). ‘Influence of Blood Inertia on Vortex Enhancement in the Wake of Plaque Deposited Arties’, Mehran University Research Journal Engineering and Technology, 30(2), 213–224.